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Workshop on

Modeling and Numerical Analysis of Nonlinear Phenomena 2025

in conjuction with

Mathematical Aspects of Continuum Mechanics 2025

(formerly **Co**ntinuum **M**echanics **Fo**cusing on **S**ingularities)

Shiinoki Cultural Complex, Garden Room
Kanazawa City, Ishikawa Prefecture, Japan

March 16 – 17, 2025

Sponsored by

What is MNANP2025?

MNANP2025 is a workshop dedicated to the Modeling and Numerical Analysis of Nonlinear Phenomena, aimed at showcasing the latest research advancements and developments in this dynamic field. The workshop will emphasize practical applications and encompass a broad range of topics, including nonlinear partial differential equations, complex fluids and materials, viscoelasticity, moving boundaries, among others. Leading experts will present their findings, while interactive sessions will facilitate collaboration and the exchange of knowledge. Participants will gain valuable insights into mathematical frameworks and computational methodologies for tackling complex challenges across various disciplines such as physics, biology, and engineering.

The workshop is organized in conjunction with CoMFoS2025, which was initiated in 1995 under the auspices of the activity group Continuum Mechanics Focusing on Singularities (CoMFoS) of the Japan Society for Industrial and Applied Mathematics (JSIAM). From April 2010, the activity group CoMFoS was renamed Mathematical Aspects of Continuum Mechanics (MACM). The recent CoMFoS meetings can be accessed here: CoMFoS2020, CoMFoS2021, CoMFoS2022, CoMFoS2023.

Acknowledgements

Date and Venue

The workshop is held from Sunday, March 16, to Monday, March 17, 2025, at the Garden Room (2F) of Shiinoki Cultural Complex, Kanazawa City, Ishikawa Prefecture, Japan.

Address: 2-chōme-1-1 Hirosaka, Kanazawa City, Ishikawa Prefecture 920-0962.

Access From JR Kanazawa Station Bus Terminal:

- Kenrokuen Exit (East Exit): Bus Stop No. 3, 6, 7, 8, 9, 10, 11
- Kanazawa Port Exit (West Exit): Bus Stop No. 5
- Get off at "Hirosaka, 21st Century Museum of Contemporary Art" (about 12 minutes) and walk about 1 minute.

Speakers

Md. Joni Alam	Kanazawa University, Japan
Annika Bach	Technische Universiteit Eindhoven, The Netherlands
Michal Beneš	Czech Technical University in Prague, Czech Republic
Giulio Giusteri	Università degli Studi di Padova, Italy
Shiro Hirano	Hirosaki University, Japan
Nicklas Jävergård	Karlstad University, Sweden
Pu-Zhao Kow	National Chengchi University, Taiwan
Ikhsan Maulidi	Kanazawa University, Japan
Kazunori Matsui	Tokyo University of Marine Science and Technology, Japan
Md. Mamun Miah	Kanazawa University, Japan
Koondanibha Mitra	Technische Universiteit Eindhoven, The Netherlands
Hiroyuki Miyoshi	The University of Tokyo, Japan
Ahmad Mohiuddin	Kanazawa University, Japan
Adrian Muntean	Karlstad University, Sweden
Petr Pauš	Czech Technical University in Prague, Czech Republic
Gilbert Peralta	University of the Philippines Baguio, Philippines
Kharisma Surya Putri	Kanazawa University, Japan
Julius Fergy Rabago	Kanazawa University, Japan
Katsuhiko Sato	University of Toyama, Japan
Makoto Sato	Kanazawa University, Japan

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Koya Sakakibara	Kanazawa University, Japan

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‡ *30-minute talk*

Day 1, 09:30 - 10:00, Muntean

Organic solar cells and mathematics of phase separation for interacting ternary mixtures under evaporation: The unexpected story of a non-local evolution system

Adrian Muntean
Department of Mathematics and Computer Science, Karlstad University, Sweden

Broadly seen, the research reported in this talk is motivated by our interest in using well-established mathematical methodologies to understand materials science aspects that we believe are responsible for the steadily increasing efficiency of organic solar cells (OSC) when harvesting solar energy [5]. In particular, as morphologies play a key role in the well-functioning of OSCs, we focus now only on their formation although our scientific curiosity has a deeper goal - unveiling the morphology's ability to transport efficiently charges.

From a more concrete point of view, being very much inspired by experimental evidence collected when processing thin films from ternary solutions made of two solutes, typically polymers, and one solvent, we study computationally the morphology formation of domains obtained in three-state systems using both a lattice (microscopic) model and a continuum (macroscopic) counterpart. The lattice-based approach relies on the Blume-Capel nearest neighbor model with bulk conservative Kawasaki dynamics (see [4]), whereas as continuum model we consider a coupled system of evolution equations (with nonlinear nonlocal drifts) that is derived as hydrodynamic limit when replacing the nearest neighbor interaction in the lattice case by a suitable Kac potential (see [1]). We explore numerically how the obtained morphology depends on the solvent content in the mixture. In particular, we study how these scenarios change when the solvent is allowed to evaporate. Essentially, we illustrate how the evaporation process affects the shape and connectivity of the evolving-in-time morphologies. Finally, we give a statement about the well-posedness of the continuum model, sketch the proof of our result (see [3] for the details), and then point out how well our finite volumes schemes are able to construct approximations of the wanted weak solution. The technical details on how our numerical scheme dissipates an approximate energy of the system can be retrieved from [2].

This is a report on recent joint work with Rainey Lyons (University of Colorado Boulder, USA), Stela Andrea Muntean (Karlstad University, Sweden), and Emilio N.M. Cirillo (La Sapienza University, Rome, Italy).

We acknowledge financial support from STINT via the project *Mathematics Bachelor Program for Efficient Computations* (nr. DD2017-6936) for promoting Karlstad-Kanazawa academic exchanges. Partial funding from the Swedish Research Council via the project "*Homogenization of Nonlinear Drift-diffusion Systems for Charge Transport through Bicontinuous Media*" (nr. 2024-05606) is much appreciated.

References

[1] Lyons, R., Muntean, S. A., Cirillo, E, N. M., Muntean, A. *A continuum model for morphology formation from interacting ternary mixtures: Simulation study of the formation and growth of patterns*, Physica D: Nonlinear Phenomena, Vol. 453 (2023), 133832.

[2] Lyons, R., Muntean, A., Nika, G., *A bound preserving energy stable scheme for a nonlocal Cahn–Hilliard equation*, Comptes Rendus Mécanique, Vol. 352 (2024), pp. 239-250.

[3] Lyons, R., Cirillo, E, N. M., Muntean, A., *Phase separation and morphology formation in interacting ternary mixtures under evaporation: Well-posedness and numerical simulation of a non-local evolution system*, Nonlinear Analysis: Real World Applications, Vol. 77 (2024), 104039.

[4] Cirillo, E, N. M., Colangeli, M., Moons, E. Muntean, A., Muntean, S. A., van Stam, J., *A lattice model approach to the morphology formation from ternary mixtures during the evaporation of one component*, The European Physical Journal Special Topics, Vol. 228 (2019), 5, 55-68.

[5] Zhang, R., Chen, H., Wang, T. et al., *Equally high efficiencies of organic solar cells processed from different solvents reveal key factors for morphology control*. Nature Energy (2024). https://doi.org/10.1038/s41560-024-01678-5.

Day 1, 10:00 - 10:30, Jävergård

Mean Field Game: Decentralized control for peak shaving and valley filling

Nicklas Jävergård, Adrian Muntean
Karlstad University, Karlstad, Sweden

Rainey Lions
Univeristy of Coloroado Boulder, Boulder, United States

In this abstract, we present our work constructing a finite volume scheme for a particular Mean Field Game model. It is concerned with a dynamic demand management model presented in [1]. The model strives to achieve peak shaving and valley filling in the power grid frequency using devices with high thermal capacity. Mean Field Games is the a field that was initiated by Larsy and Lions in their seminal paper [2]. It has since been developed and investigated by many authors and applied to many different settings such as finance, crowd dynamics and frequency and power selection for cellular communication.

Acknowledgments

We thanks the Swedish Energy Agency's project Solar Electricity Research Centre (SOLVE) with grant number 52693-1 for their support.

References

[1] F. Bagagiolo, D.Bauso, *Mean-Field Games and Dynamic Demand Management in Power Grids*, Dynamic Games and Applications, 4, 2014.

[2] Lasry, J.-M. Lions, P.-L., *Mean Field Games*, Japanese Journal of Mathematics, 2007.

Day 1, 10:45 - 11:15, Bach

Anisotropic and isotropic free-discontinuity functionals as Γ-limits of discrete functionals

Annika Bach
Eindhoven University of Technology, The Netherlands

This talk is concerned with the emergence of anisotropies in a variational coarse-graining procedure of discrete Ambrosio-Tortorelli type functionals. In a continuum setting, the latter provide an elliptic approximation in terms of Γ-convergence of the so-called Mumford-Shah functional, an isotropic free discontinuity functional which is frequently used in image segmentation. Discretisations of the Ambrosio-Tortorelli functionals might instead converge (as both the lattice spacing and the elliptic approximation parameter tend to zero) to an anisotropic free-discontinuity functional depending on the underlying lattice structure. This happens in particular for discretisations on a periodic lattice when the lattice spacing is of the same order as the elliptic parameter. In contrast, we shall see that discretisations on suitable random lattices still converge to the Mumford-Shah functional in this regime.

The talk is based on joint works with Andrea Braides (Trieste/ Roma), Marco Cicalese (München), Matthias Ruf (Augsburg), and Caterina Ida Zeppieri (Münster).

Day 1, 11:15 - 11:30, Miah

Numerical study on crack path by fracture phase field model with unilateral contact condition and energy dissipation identity

Md. Mamun Miah[1], Ounissi Oussama[1], Sayahdin Alfat[1] and Masato Kimura[2]

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[2]Faculty of Mathematics and Physics, Kanazawa University, Kanazawa, 920-1192, Japan.

Fracture problems in modern science and technology are prevalent and severe issues nowadays. For the numerical simulation of crack propagation, the fracture phase field model (PFM) is commonly used in fracture mechanics. We examine the numerical simulation for crack propagation due to the various scenarios, such as mode I crack propagation, mixed mode crack propagation, and compression in the domain with inclined and horizontal cracks, by using the finite element method (FEM) for PFM. Here, we applied PFM to investigate the two-dimensional and three-dimensional problems in the original model and the model with unilateral contact condition. Using the numerical technique FEM, in every instance, we observed the realistic fracture propagation feature. For the case of unrealistic, we mentioned the PFM with unilateral contact condition. Here, we also proposed the model with a unilateral contact condition to avoid unrealistic cases of crack propagation. Using both cases, we noticed that mode I and crack propagation due to compressing in the domain with horizontal crack yield realistic results. The model with unilateral contact condition gave us a realistic crack path for the mixed mode, whereas the original PFM gave us a branching crack path, which is not possible. For crack propagation due to compressing in the domain with inclined crack, the original is not realistic, but the crack profile is realistic in the model with unilateral contact condition. We also discussed and theoretically proved the energy dissipation identity, more commonly known as the waste energy identity for both the original and unilateral contact condition. This identity helps to formulate fracture criteria like Griffith's criterion, which states that a crack grows when the energy release rate exceeds the toughness of the fracture. Thus, we may conclude that compared to the original model, crack propagation resulting from unilateral contact circumstances produces more realistic findings. In some circumstances, we also discuss the driving force profile and surface energy of the original and unilateral contact condition. Here, we used FreeFem++ software for numerical simulation and symmetric mesh generation with adaptation.

Day 1, 13:30 - 14:00, Kow

Stability of linear inverse problems: a singular value decomposition approach

Pu-Yun Kow
Department of Bioenvironmental System Engineering, National Taiwan University

Pu-Zhao Kow
Department of Mathematical Sciences, National Chengchi University

Mikko Salo
Department of Mathematics and Statistics, University of Jyväskylä

Sen Zou
School of Mathematical Science, Fudan University

The main theme of this talk is to explain a general principle that the stability of linear inverse problem can be known from the decay asymptotic of single values. For simplicity, we consider the problem of recovering the density of a Herglotz wave function, based on [1]. We also exhibit some results in [2], which is a part of Helsinki Speech Challenge 2024 [3].

References

[1] Pu-Zhao Kow, Mikko Salo and Sen Zou, *Increasing resolution and instability for linear inverse scattering problems*, arXiv preprint, 2024. `https://doi.org/10.48550/arXiv.2404.18482`

[2] Pu-Yun Kow and Pu-Zhao Kow, *An efficient light-weighted signal reconstruction method consists of Fast Fourier Transform and Convolutional-based Autoencoder*, arXiv preprint, 2024. `https://doi.org/10.48550/arXiv.2501.01650`

[3] Martin Ludvigsen, Elli Karvonen, Markus Juvonen and Samuli Siltanen, *Helsinki Speech Challenge 2024*, arXiv preprint, 2024. `https://doi.org/10.48550/arXiv.2406.04123`.

Day 1, 14:00 - 14:30, Rabago

Localization of tumor through a non-conventional numerical shape optimization technique

Julius Fergy Rabago
Kanazawa University, Japan

In this presentation, we introduce a new method for estimating the shape and location of an embedded tumor using shape optimization. Our approach applies the coupled complex boundary method [1, 2, 3], which reformulates the problem–defined by a measured temperature profile and corresponding flux (e.g., from infrared thermography)–as a complex boundary value problem with a complex Robin boundary condition. This effectively handles the over-specified nature of the problem.

We identify the tumor by optimizing a cost function based on the imaginary part of the solution. To compute the shape derivative, we perform shape sensitivity analysis. We also propose an iterative algorithm that uses the Riesz representative of the gradient to numerically determine the tumor shape via the finite element method. Numerical examples confirm the accuracy and effectiveness of our method. The results are based on [4].

References

[1] J. F. T. Rabago. On the new coupled complex boundary method in shape optimization framework for solving stationary free boundary problems. *Math. Control Relat. Fields*, 13(4):1362–1398, 2023.

[2] J. F. T. Rabago and H. Notsu. Numerical solution to a free boundary problem for the Stokes equation using the coupled complex boundary method in shape optimization setting. *Appl. Math. Optim.*, 89:Art. 2, 2024.

[3] J. F. T. Rabago, L. Afraites, and H. Notsu. Detecting immersed obstacle in Stokes fluid flow using the coupled complex boundary method. *SIAM J. Control & Optim.*, to appear, 2025.

[4] J. F. T. Rabago. Localization of tumor through a non-conventional numerical shape optimization technique. February, 2025. `arXiv:2502.20656`.

Day 1, 14:45 - 15:15, Mitra

Degenerate & singular mixed dimensional diffusion systems
A journey from modelling, and numerical methods, to well-posedness

K. Mitra
Eindhoven University of Technology, the Netherlands

J.A. Geurts[1], R.K.H. Smeets[2], A. Javed[3], S. Sonner[1], I.S. Pop[3]
[1] Radboud University, the Netherlands
[2] University of Amsterdam, the Netherlands
[3] Hasselt University, Belgium

Nonlinear, degenerate, and coupled (mixed-dimensionally) systems arise in various applications of societal relevance such as biofilm growth, reactive transport in fractured porous media, and cellular biology. For such problems, the biomass or the main-substrate is restricted to a lower dimensional manifold Γ embedded in a domain Ω in $\mathbb{R}^d$, and the evolution of the biomass density/substrate concentration exhibits degenerate and singular diffusion behaviour. The other equations (for nutrients/reagents) are defined on the bulk Ω, and are of linear advection-reaction-diffusion type.

In our analysis, we first propose a backward Euler time-discretisation of the problem where the reactive terms coupling the equations are estimated semi-implicitly, and thus, the equations are dimensionally decoupled and can be solved sequentially. A bulk-surface finite element method is used to discretise the system in space. Then, an iterative linearisation algorithm is proposed to solve the fully-discrete nonlinear problem on discretised Γ. It is proven that the iterations converge unconditionally even for degenerate/singular cases and for curved Γ, thus, showing existence of solution of the fully-discrete nonlinear problem. Then, for flat Γ, the convergence of the fully-discrete solutions to the time-discrete solution is shown and order of convergence with respect to mesh-size is estimated. Properties such as well-posedness, boundedness, and positivity of the time-discrete solutions are proven, and the existence of the time-continuous solutions is shown by passing the time-step size to zero. Uniqueness is shown using a contraction argument. Additional properties such as regularity and finite-time blow-up are investigated. Numerical results validate the theoretical predictions. They demonstrate that the iterative solver and the discretization are extremely robust and efficient compared to existing alternatives.

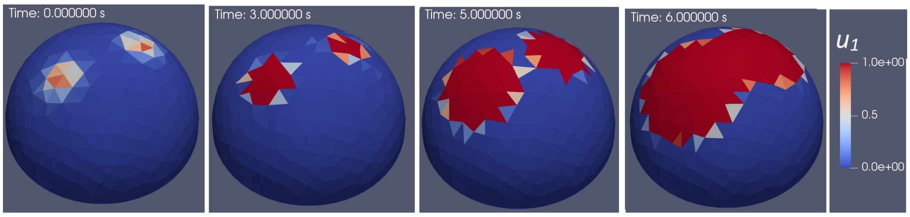

Figure 1: Numerical study of a nonlinear mixed-dimensional system (fully-discrete): two biofilm patches that stick to the surface of a sphere interact and grow by consuming nutrients (N_2 and O_2) from the interior.

References

[1] K. Mitra, & S. Sonner (2023). Well-posedness and properties of nonlinear coupled evolution problems modelling biofilm growth. arXiv:2304.00175.

[2] R.K.H. Smeets, K. Mitra, S. Sonner, & I.S. Pop (2024). Robust time-discretisation and linearisation schemes for singular and degenerate evolution systems modelling biofilm growth. arXiv:2404.00391v2.

Day 1, 15:15 - 15:45, Miyoshi

Leapfrogging effect of four point vortices in a periodic parallelogram

Hiroyuki Miyoshi
The University of Tokyo, Japan

Two vortex rings with the same axis exhibit a characteristic behavior known as the leapfrogging effect, first observed by Helmholtz in 1867 [1]. When two ring-shaped vortex filaments share the same center, one ring leaps over the other. If the two vortex rings are arranged in an ideal configuration, this motion continues periodically. This phenomenon is referred to as the leapfrogging effect. Although this effect has been studied for over 150 years, questions regarding its stability, bifurcation, and the existence of boundaries remain open.

Galantucci et al. (2021) numerically investigated the motion of four vortices within a bounded channel and discovered an intriguing phenomenon termed "image-driven" leapfrogging [3]. In their study, the vortex dynamics were classified into four distinct categories: (i) no leapfrogging, (ii) forward leapfrogging, (iii) image-driven leapfrogging, and (iv) periodic orbits. Notably, cases (iii) and (iv) were found to be unique to such confined geometries.

In this presentation, we present a new observation regarding the dynamics of four vortices in a periodic parallelogram. This study extends the work of Krishnamurthy and Sakajo (2023) [2] by introducing the complex modulus of the prime function developed by Crowdy (2020). Our numerical approach reveals critical parameter values necessary for the existence of the leapfrogging effect in periodic parallelograms.

References

[1] Helmholtz, H von 1867 Lxiii. on integrals of the hydrodynamical equations, which express vortex-motion. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 33 (226), 485–512.

[2] Krishnamurthy, V. S. and Sakajo, T. 2023 The N-vortex problem in a doubly periodic rectangular domain with constant background vorticity. Physica D: Nonlinear Phenomena 448, 133728.

[3] Galantucci, Luca, Sciacca, Michele, Parker, Nick G, Baggaley, Andrew W & Barenghi, Carlo F 2021 Classical and quantum vortex leapfrogging in two-dimensional channels. Journal of Fluid Mechanics 912, A9.

Day 1, 16:00 - 16:15, Maulidi

A generalized averaging of a function and characterization of Sobolev spaces on the sphere

Ikhsan Maulidi, Hiroshi Ohtsuka
Graduate School of Natural Science and Technology, Kanazawa University, Japan

We introduce a new characterization of Sobolev spaces on the sphere, building upon the work of Barceló et al. (2020). Our approach extends their square function to characterize Sobolev spaces. We examined the weight and range of the averaging in the square function and demonstrated how the domain of averaging can be restricted to a local coordinate system on the sphere.

Let $T \in (0, \pi]$ and $\rho \in L^\infty(0,T)$ with $\rho \geq 0$ a.e. and $\int_0^T \rho(\theta)d\theta > 0$. For an integrable function f on $\mathbf{S}^{d-1}$ and $t \in (0,T]$, the generalized averaging of f around a spherical cap $C(\xi, t)$ is defined as

$$A_t^\rho f(\xi) := z_t^{-1} \int_{C(\xi,t)} f(\theta, \tau')\rho\left(\frac{T}{t}\theta\right) d\sigma,$$

where z_t is chosen to satisfy $z_t^{-1} \int_{C(\xi,t)} \rho\left(\frac{T}{t}\theta\right) d\sigma = 1$.

The generalized square function is defined as

$$S_\alpha^{\rho,T}(f)^2(\xi) := \int_0^T \left| \frac{A_t^\rho f(\xi) - f(\xi)}{t^\alpha} \right|^2 \frac{dt}{t}.$$

The main theorem is as follows : For $0 < \alpha < 2$, the following are equivalent:

(1) $f \in H^\alpha\left(\mathbf{S}^{d-1}\right)$.
(2) $f \in L^2(\mathbf{S}^{d-1})$ and $S_\alpha^{\rho,T}(f) \in L^2(\mathbf{S}^{d-1})$.

References

[1] Kendall Atkinson and Weimin Han, Spherical harmonics and approximations on the unit sphere: an introduction, Lecture Notes in Math., 2044, 2012, Springer, Heidelberg, 2012, x+244 pp.

[2] Roc Alabern, Joan Mateu, Joan Verdera, A new characterization of Sobolev spaces on $\mathbf{R}^n$, Math. Ann. **354** (2012) no.2, 589 – 626.

[3] Juan Antonio Barceló, Teresa Luque, Salvador Pérez-Esteva, Characterization of Sobolev spaces on the sphere, J. Math. Anal. Appl. **491** (2020) no.1, 124240, 23 pp.

[4] Ikhsan Maulidi & Hiroshi Ohtsuka. (2025). A generalized averaging of a function and characterization of Sobolev spaces on the sphere, The Science Reports of Kanazawa University **68** (2025), 1 – 9.

Day 1, 16:15 - 16:45, Peralta

Weak and Very Weak Solutions for Viscous Non-Isothermal Binary Fluids

Gilbert Peralta
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University of the Philippines Baguio, Philippines

We present a system of nonlinear partial differential equations governing the dynamics of non-isothermal, viscous, and incompressible binary fluid flows on two-dimensional domains. This system couples the Cahn–Hilliard equation for non-equilibrium phase transitions and the Oberbeck–Boussinesq system in thermohydraulics accounting for surface tension due to capillary action. Suitable definitions for weak and distributional solutions will be addressed for low-regular source functions and initial data in some interpolation spaces. These solution concepts cover the case of measure-valued sources. While the weak solutions are differentiable in the Sobolev sense, the distributional solutions, in general, are not. For the Navier–Stokes part, the solutions we consider here belong to Serrin's class; hence, uniqueness is to be expected. We also present a space-time version of the classical de Rham's theorem for the existence of the associated pressure. The essential tools employed in the analysis are the maximal parabolic regularity for the linearized system and the well-posedness of the nonlinear part, with the solution of the linearized dynamics as the frozen coefficients. Differentiability properties of the operator that maps the sources and initial data to the solutions and higher temporal-integrability of solutions will be mentioned.

Day 2, 09:30 - 10:00, Giusteri

Mathematical modeling of viscoelastic materials

Giulio G. Giusteri
Department of Mathematics, University of Padua, Italy

In this talk, I will present a class of continuum mechanical models aimed at describing the behavior of viscoelastic materials by incorporating concepts originated in the theory of solid plasticity in a fluid mechanics context [1]. Within this class, even a simple model with constant material parameters is able to qualitatively reproduce a number of experimental observations in both simple shear and extensional flows, including linear viscoelastic properties, the rate dependence of steady-state material functions, the stress overshoot in incipient shear flows, and the difference in shear and extensional rheological curves.

The constitutive models are based on a logarithmic relation between the elastic strain measure and the stress tensor and on evolution equations for a local representative of the elastically-relaxed strain state. The mathematical analysis of such tensorial transport equations leads to the definition of the notion of charted weak solutions [2]. These are based on non-standard a priori estimates that involve both viscous and plastic energy dissipation. After outlining the main aspects of the theoretical analysis, I will discuss issues related to the numerical solution of the viscoelastic evolution equations within the framework of stabilized mixed finite elements [3].

References

[1] M. A. H Alrashdi, G. G. Giusteri, *Evolution of local relaxed states and the modeling of viscoelastic fluids*, Phys. Fluids, 36, 093129, 2024.

[2] G. Ciampa, G. G. Giusteri, A. G. Soggiu, *Viscoelasticity, logarithmic stresses, and tensorial transport equations*, Math. Meth. Appl. Sci. 48, 2934–2953, 2025.

[3] N. Dash, R. Codina and G. G. Giusteri, *Stabilized mixed elements for viscoelastic fluid models*, in preparation.

Day 2, 10:00 - 10:30, Matsui

Projection Scheme for Elastoplastic Problems with Kinematic Hardening

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Department of Logistics and Information Engineering,
Tokyo University of Marine Science and Technology, Japan

Yoshiho Akagawa
National Institute of Technology (KOSEN), Gifu College, Japan

When a large force is applied to metallic materials, plastic deformation occurs, remaining even after the force is removed. In materials subjected to cyclic loading, strain hardening is observed. One effective description of this hardening behavior is the kinematic hardening rule. In the kinematic hardening rule, the constraint set bounded by the yield surface is considered to translate in stress space as plastic deformation progresses. In this presentation, we propose a new numerical scheme (based on [1]) for elastoplastic models incorporating this kinematic hardening rule. We prove that the solution is stable under appropriate norms, even for cases where the relationship between plastic strain and the center of the constraint set (back stress) is nonlinear (Armstrong-Frederick model). Moreover, this stability leads to the existence of a solution to the original problem.

References

[1] Y. Akagawa and K. Matsui, Projection scheme for a perfect plasticity model with a time-dependent constraint set, *Journal of Mathematical Analysis and Applications*, 542 (2), 128838, 2025.

Day 2, 10:45 - 11:15, Paus

Numerical simulation of dislocation multiple cross-slip

Petr Pauš, Michal Beneš, Miroslav Kolář
Czech Technical University in Prague

Our contribution deals with the phenomenon in material science called multiple cross-slip of dislocations in slip planes. The numerical model is based on a mean curvature flow equation with additional forcing terms included [1]. The curve motion in 3D space is treated using our tilting method, i.e., mapping of a 3D situation onto a single plane where the curve motion is computed. The physical forces acting on a dislocation curve are evaluated in the 3D setting.

References

[1] Kolář, Miroslav, et al., *Improving method for deterministic treatment of double cross-slip in FCC metals under low homologous temperatures*, Computational Materials Science 189, 2021.

Day 2, 11:15 - 11:30, Alam

Estimating the average free boundary velocity in the Hele-Shaw problem using a Berger-Brezis-Rogers like numerical scheme

Md. Joni Alam
Graduate School of Natural Science and Technology, Kanazawa University, Kanazawa, 920-1192, Japan

The Hele-Shaw problem is a popular model of the flow of an incompressible fluid between two closely spaced parallel plates, known as the Hele-Shaw cell. In its classical form, the Hele-Shaw problem is homogeneous as it does not involve a time-dependent coefficient. However, we focus on a Hele-Shaw problem in an inhomogeneous medium, where the coefficient in the free boundary velocity depends on both space and time. The homogenization behavior in such a situation exhibits interesting effects, and finding the average velocity of free boundary movement is an interesting problem. We develop a novel BBR-like [Berger-Brezis-Rogers, 1979] scheme for the Hele-Shaw problem with a time-dependent coefficient. To discretize in time, the BBR-like scheme is used, while quadtree and octree structures are applied for spatial discretization. We use adaptive mesh refinement with quadtree structures in two dimensions and octree structures in three dimensions to make computing more efficient. We evaluate the performance of our method and provide several examples. Numerical experiments demonstrate that our method yields a more robust and precise estimation of average free boundary velocity in comparison with the results presented in [Palupi-Pozar, 2018].

This is a joint work with Norbert Pozar from the Faculty of Mathematics and Physics, Institute of Science and Engineering, Kanazawa University, Kanazawa, 920-1192, Japan.

References

[1] Berger, Alan E., Brézis, Haïm, and Rogers, Joel C. W. *A numerical method for solving the problem* $u_t - \Delta f(u) = 0$. RAIRO Anal. Numér., **13**(4), 297–312, 1979.

[2] Palupi, I. and Požár, N. *An efficient numerical method for estimating the average free boundary velocity in an inhomogeneous Hele-Shaw problem.* Sci. Rep. Kanazawa Univ., **62**, 69–86, 2018.

Day 2, 13:30 - 14:00, KSato

A cell membrane model that reproduces cortical flow-driven cell migration and collective movement

Katsuhiko Sato
Science, University of Toyama, Japan

Many fundamental biological processes are dependent on cellular migration. Although the mechanical mechanisms of single-cell migration are relatively well understood, those underlying migration of multiple cells adhered to each other in a cluster, referred to as cluster migration, are poorly understood [1]. A key reason for this knowledge gap is that many forces-including contraction forces from actomyosin networks, hydrostatic pressure from the cytosol, frictional forces from the substrate, and forces from adjacent cells contribute to cell cluster movement, making it challenging to model, and ultimately elucidate, the final result of these forces. In this talk, I provide a two-dimensional cell membrane model that represents cells on a substrate with polygons and expresses various mechanical forces on the cell surface, keeping these forces balanced at all times by neglecting cell inertia [2]. The model is discrete but equivalent to a continuous model if appropriate replacement rules for cell surface segments are chosen. When cells are given a polarity, expressed by a direction-dependent surface tension reflecting the location dependence of contraction and adhesion on a cell boundary, the cell surface begins to flow from front to rear as a result of force balance. This flow produces unidirectional cell movement, not only for a single cell but also for multiple cells in a cluster, with migration speeds that coincide with analytical results from a continuous model. Further, if the direction of cell polarity is tilted with respect to the cluster center, surface flow induces cell cluster rotation. The reason why this model moves while keeping force balance on cell surface (i.e., under no net forces from outside) is because of the implicit inflow and outflow of cell surface components through the inside of the cell. An analytical formula connecting cell migration speed and turnover rate of cell surface components is presented.

References

[1] Friedl, P., and Gilmour, D., *Collective cell migration in morphogenesis, regeneration and cancer*, Nat. Rev. Mol. Cel. Biol., 10 (7), 445-457, (2009).

[2] Sato K, *A cell membrane model that reproduces cortical flow-driven cell migration and collective movement*, Front. Cell Dev. Biol., 11:1126819, (2023).

Day 2, 14:00 - 14:30, MSato

Tiling mechanisms of the compound eye through forces

Makoto Sato, Steven R. Davis, and Ting Zheng
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Tilling patterns are found in many biological structures such as insect compound eyes, columnar structures in the brain, and lobules in the liver. Among them, hexagonal tilling is dominant probably because it is superior to the other tilling patterns in terms of physical properties such as structural strength, boundary length and space filling. The compound eye of fruit fly is made from ommatidial eye units showing regular hexagonal pattern and is an ideal model to understand the mechanism of tiling. Interestingly, it also shows tetragonal pattern in some mutant backgrounds. Here, we propose a universal tiling mechanism of the compound eye. Voronoi diagram is often used to equally divides multiple areas according to the distance from the center of each area. We found that the wildtype hexagonal pattern and mutant tetragonal pattern perfectly fit with Voronoi diagram. Incorporating the tissue-wide tension along the dorsal-ventral axis observed in vivo, the hexagonal pattern is transformed to the tetragonal pattern. How does ommatidial shape obey the geometrical Voronoi patterns? To answer this question, we focused on mutant eyes, in which the tilling pattern becomes stochastic. Surprisingly, such a stochastic ommatidial pattern also fit with Voronoi diagram except for occasional mismatching found in smaller and larger ommatidia. The growth of ommatidia, which is largely affected by the number of cells within individual ommatidia, may play critical roles. We demonstrated that in the presence of the differential growth of ommatidia, the result of weighted Voronoi diagram nicely fits with the mutant pattern. Thus, physical stretch of the eye tissue and the concentric growth of cells co-operatively determine the ommatidial tiling patterns. Physical entity that promotes the cellular growth and molecular machinary that control the cellular force response will be discussed.

References

[1] T. Hayashi, T. Tomomizu, T. Sushida, M. Akiyama, S-.I. Ei, and M. Sato, *Tiling mechanisms of the Drosophila compound eye through geometrical tessellation.* Current Biology 32, pp. 2101-2109 (2022).

[2] H. Togashi, S. R. Davis, and M. Sato, *From soap bubbles to multicellular organisms: Unraveling the role of cell adhesion and physical constraints in tile pattern formation and tissue morphogenesis.* Developmental Biology 506, pp. 1-6 (2024).

Day 2, 14:45 - 15:15, Hirano

Stochastic sources for earthquake ground motion simulations

Shiro Hirano
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Ground motion simulations are essential for risk assessment in earthquake science and engineering. We need appropriate models for both a propagator from the earthquake source to the Earth's surface and a kinematic or dynamic source model for the simulations. The source is a time series of co-seismic slip velocity along a fault that is a rupture plane in the Earth's crust. Damages observed on the Earth's surface depend on the frequency content in the time series, where power especially in ~ 1 Hz strongly affects them. Therefore, a plausible source model plays a key role. Still, the fault motion has shown diversity even for almost the same magnitude events, and a deterministic description of the source is problematic.

We have suggested a model for the earthquake sources based on a stochastic differential equation[1]. The time series $S(t)$ is given by a convolution of two Bessel processes as

$$S(t) = \int_0^t X^{(1)}(s)\, X^{(2)}(s-t)\, ds,$$
$$dX^{(i)}(t) = -\frac{dt}{2X^{(i)}(t)} + dB_t^{(i)},\ (i = 1, 2)$$

where $B_t^{(i)}$ is the standard Brownian motion. The convolution satisfies multiple empirical laws in earthquake seismology: their power spectral density follows f^{-4} (f: frequency), their expectation follows t^2 (t: time since ignition), and the probability density follows $\left|\int_0^\infty S(t)\, dt\right|^{2/3}$.

We briefly review the role of the stochastic model for seismic risk assessment. The model outputs can be used as inputs for seismic ground motion simulations as follows:

$$v(t) = (G * S)(t),$$

where v is the ground velocity, and G is the propagator. We discuss how the simulated ground velocity is uncertain due to the stochasticity of the source.

References

[1] Hirano, S., *Source time functions of earthquakes based on a stochastic differential equation*, Sci. Rep., (2022) 12:3936, doi:10.1038/s41598-022-07873-2

Day 2, 15:15 - 15:30, Mohiuddin

Modelling cleaved cell proliferation by dividing swarmalators

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Ayumi Ozawa
Japan Agency for Marine-Earth Science and Technology, Japan

Matthew Smart, Hayden Nunley
Flatiron Institute, United States

During early embryonic development of many species, a series of cell divisions generates, from one cell, a cluster of tens or hundreds of cells. This often occurs by cell divisions with no growth, which we refer to as cleaving divisions, which are often initially synchronous and become progressively desynchronized. Here, we model the proliferation dynamics of cleaved cells. These cells divide in a fixed domain that has constant volume. We assume that the geometry of the cleaved cells is given by a Voronoi tessalation in 2D. Towards investigating the apparent synchrony of division events, we model the cells as oscillators with spatial degrees of freedom that are coupled to their neighbours, exploring the effect of coupling on the resulting divisions. Here, we assume the so-called Hertwig's rule which states that cells divide along their longest axis [1, 2] to model the division. We thus have a model of spatially coupled oscillators which proliferate as they complete full cycles, constituting a biologically-inspired generalization of the classic swarmalator model [3]. We observe that our implementation of these assumptions yields a model that qualitatively behaves like the biology. Our future work concerns rigorous results for finite-time blow up of divisions, dynamical systems analysis of local synchronization, connecting lineage statistics of simulations to biological experiments.

References

[1] Hertwig, O. (1884). *Welchen Einfluß übt die Schwerkraft auf die Theilung der Zellen? (No. 2). G. Fischer.*

[2] Strauss, B., Adams, R. J., & Papalopulu, N. (2006). A default mechanism of spindle orientation based on cell shape is sufficient to generate cell fate diversity in polarised *Xenopus* blastomeres. *Development*, 133(17), 3333-3341.

[3] O' Keeffe, K. P., Hong, H., & Strogatz, S. H. (2017). Oscillators that sync and swarm. *Nature Communications*, 8(1), 1504.

Day 2, 15:30 - 15:45, Putri

Lagrangian–Galerkin Moving Mesh Method

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Niklas Kolbe
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Hirofumi Notsu
Faculty of Mathematics and Physics, Kanazawa University, Japan

In this research, our goal is to accurately capture high-concentration phenomena that lead to sharp spike patterns, such as those observed in cancer cell aggregation. These problems are typically modeled by convection-dominated PDEs, which require careful domain discretization to achieve high-resolution results. In recent years, adaptive mesh methods and optimal transport techniques have gained popularity as effective approaches for handling such challenges.

In this talk, based on joint work with T. Mizuochi, N. Kolbe, and H. Notsu (see [1]), we introduce the **Lagrangian–Galerkin Moving Mesh Method (LGMM)** as a low-cost computational alternative for handling high-concentration phenomena. Here, LGMM extends the mass-conservative Lagrange–Galerkin (LG) framework, a powerful method for convection-dominated problems, into a moving mesh setting. In this approach, mesh nodes are dynamically moved to follow the underlying flow, while maintaining the mass conservation property inherent in fixed-mesh LG methods.

By carefully selecting time increments, we can prevent mesh overlap and degeneration, ensuring a stable and accurate solution process. Moreover, for linear elements, we establish an optimal error estimate in the $\ell^\infty(L^2) \cap \ell^2(H_0^1)$ norms. This result is derived from new bounds on a time-dependent interpolation operator developed in our work. Through numerical experiments, we validate our theoretical findings and highlight several advantages of our moving mesh approach compared to the standard fixed-mesh Lagrangian-Galerkin method.

References

[1] Putri, K.S., Mizuochi, T., Kolbe, N., Notsu, H., *Error Estimates for First- and Second-Order Lagrange – Galerkin Moving Mesh Schemes for the One-Dimensional Convection – Diffusion Equation.* Journal of Scientific Computing 101, 2024.

Day 2, 16:00 - 16:30, Benes

Applications of Planar Curve Dynamics

Michal Beneš
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In this contribution, we investigate the motion of planar curves with velocity or acceleration given by curvature and local or non-local force (see [1]). It is treated by the parametric method with the velocity decomposed into the normal and tangent directions. We admit scalar quantities to be transported along the curves by diffusion and influenced by mutual interaction with the curve. The evolution law is given by a system of degenerate parabolic equations for which the local existence and uniqueness of solution can be obtained (see [2]). Numerical discretization can be constructed using the method of flowing finite volumes enhanced by redistribution ensuring uniformity of nodes along the curve. We demonstrate behavior of the solution on computational studies related to applications in the dislocation dynamics in the crystalline structure of materials, crystal boundary motion, or electric signal spreading in excitable media (see e.g. [3]).

References

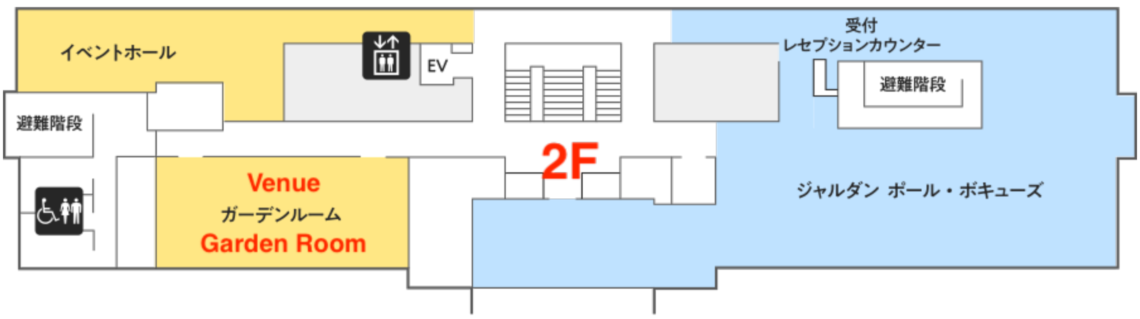
[1] Kolář M, Beneš M, Ševčovič D, *Area Preserving Geodesic Curvature Driven Flow of Closed Curves on a Surface*, Discrete Continuous Dynamical Systems B, Volume 22, Issue 10, 3671–3689, 2017.

[2] Beneš M, Kolář M, Ševčovič D, *Curvature driven flow of a family of interacting curves with applications*, Math. Meth. Appl. Sci., Volume 43, 4177–4190, 2020.

[3] Suchomelová M, Beneš M, Kolář M, *Computational aspects of hyperbolic curvature flow*, Applied Mathematics and Computation, Volume 495, 129301, 2025.

Venue

The venue of the workshop/conference is the Garden Room, located on the second floor of the Shiinoki Cultural Complex.

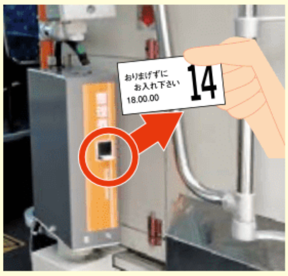


How to Ride Local-Line Buses

Enter the bus from the rear door.
Pay the fare when you get off.

Be sure to confirm the destination shown on the front and the side of the bus.

1 **Take a numbered ticket (called "Sei-ri-ken") from the machine when getting on the bus.**
Please note that numbered tickets ("Seiri-ken") are not issued on Kanazawa Loop Buses and Kanazawa Flat Buses. Simply get on those buses without taking a ticket.



Pay the bus fare when getting off. This numbered ticket shows the bus stop where you got on, and is required to pay the fare.
*On Kanazawa Flat Buses, pay the fare when you get on.

2 **After your stop is announced and shown on the display at the front of the bus, press the Request Stop button to let the driver know that you will get off at the next stop.**
The button will light up.

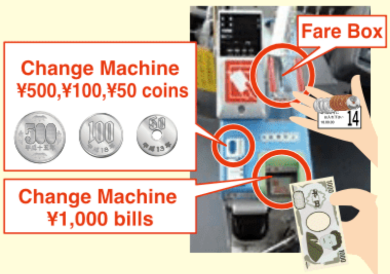
3 **Confirm the fare shown on the display.**

The child fare (for elementary school students ages 6-12) is half the adult fare (rounded up to the nearest ¥10).

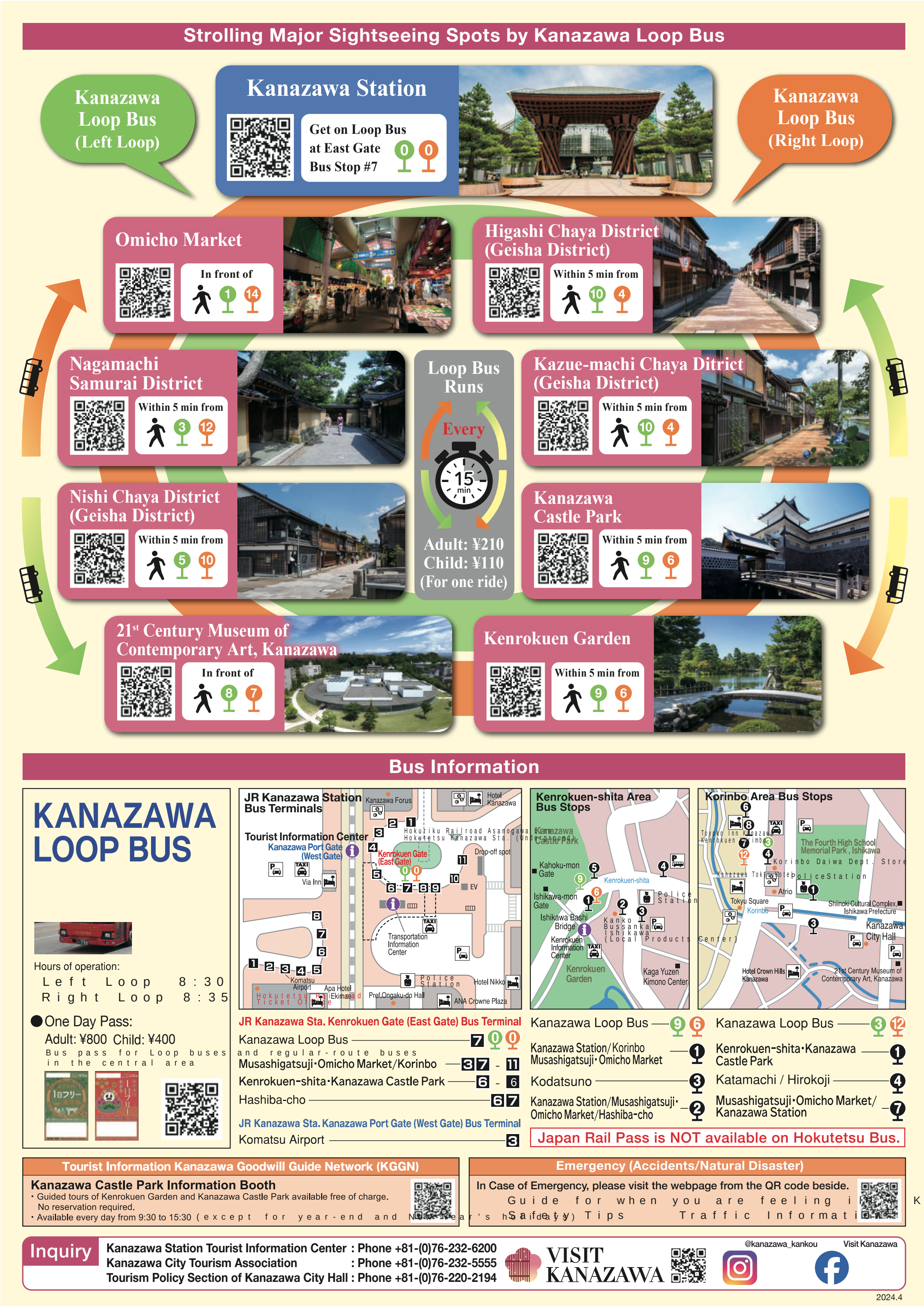
4 **When getting off, put both the fare and the numbered ticket in the fare box next to the driver.**

Since change is not given, please use the change machine to put the exact fare in the fare box.

¥10,000, ¥5,000, and ¥2,000 bills cannot be changed.



Kanazawa Local Tourist Map



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Kanazawa Loop Bus (Left Loop)

Kanazawa Loop Bus (Right Loop)

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Omicho Market
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Higashi Chaya District (Geisha District)
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Loop Bus Runs Every 15 min
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(For one ride)

Kazue-machi Chaya District (Geisha District)
Within 5 min from 10 4

Nishi Chaya District (Geisha District)
Within 5 min from 5 10

Kanazawa Castle Park
Within 5 min from 9 6

21st Century Museum of Contemporary Art, Kanazawa
In front of 8 7

Kenrokuen Garden
Within 5 min from 9 6

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Hours of operation:
Left Loop 8:30
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● One Day Pass:
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JR Kanazawa Station Bus Terminals

JR Kanazawa Sta. Kenrokuen Gate (East Gate) Bus Terminal
Kanazawa Loop Bus — 7 0 0
Musashigatsuji·Omicho Market/Korinbo — 3 7 - 11
Kenrokuen-shita·Kanazawa Castle Park — 6 - 6
Hashiba-cho — 6 7

JR Kanazawa Sta. Kanazawa Port Gate (West Gate) Bus Terminal
Komatsu Airport — 3

Kenrokuen-shita Area Bus Stops
Kanazawa Loop Bus — 9 6
Kanazawa Station/Korinbo Musashigatsuji·Omicho Market — 1
Kodatsuno — 3
Kanazawa Station/Musashigatsuji·Omicho Market/Hashiba-cho — 2

Korinbo Area Bus Stops
Kanazawa Loop Bus — 3 12
Kenrokuen-shita·Kanazawa Castle Park — 1
Katamachi / Hirokoji — 4
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- Guided tours of Kenrokuen Garden and Kanazawa Castle Park available free of charge. No reservation required.
- Available every day from 9:30 to 15:30 (except for year-end and New Year's holidays)

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