CoMFoS22 Mathematical Aspects of Continuum Mechanics 2022

December 6 - 7, 2022 · 17:00-20:00 (JST) = 09:00-12:00 (CET) · via Zoom

Schedule Overview

Day 1 (JST)	Speaker
17:00 - 17:30	Keita Yoshioka
17:30 - 18:00	Victor A. Kovtunenko
18:00 - 18:30	Sayahdin Alfat
18:30 - 19:00	Keigo Wada
19:00 - 19:30	Julius Fergy Rabago
19:30 - 20:00	Pavel Strachota

Day 1 (CET)	Speaker
09:00 - 09:30	Keita Yoshioka
09:30 - 10:00	Victor A. Kovtunenko
10:00 - 10:30	Sayahdin Alfat
10:30 - 11:00	Keigo Wada
11:00 - 11:30	Julius Fergy Rabago
11:30 - 12:00	Pavel Strachota

Day 2 (JST)	Speaker
17:00 - 17:30	Deepjyoti Goswami
17:30 - 18:00	Giulio G. Giusteri
18:00 - 18:30	Ryohei Seto
18:30 - 19:00	Kohji Ohtsuka
19:00 - 19:30	Takashi Nakazawa

Day 2 (CET)	Speaker
09:00 - 09:30	Deepjyoti Goswami
09:30 - 10:00	Giulio G. Giusteri
10:00 - 10:30	Ryohei Seto
10:30 - 11:00	Kohji Ohtsuka
11:00 - 11:30	Takashi Nakazawa

Day 1

S. = Schedule · **P.** = Presenter · **T.** = Title · **A.** = Abstract

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S. Day 1 17:00-17:30 (JST) = 09:00-09:30 (CET)

P. Keita Yoshika

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T. A variational phase-field model for hydraulic fracturing

A. Since their inception in late 90's, the phase-field models of fracture simulation have steadily gained popularity. One of the appeals is its ability to handle complex topologies with unknown crack paths in relatively coarse meshes as well as multiple-crack interaction, which is a fundamental requirement for the numerical simulation of hydraulic fracturing in complex situations and is technically more difficult to achieve with many other methods. In this talk, we will first describe the construction of a phase-field based coupled hydromechanical reservoir simulator. We will then revisit the problem of a single hydraulic fracture propagating in an infinite impermeable medium in order to verify the computation of fracture width and fracture pressure from the phase-field model.Finally we will show how a phase-field description of a system of cracks can be leveraged to model flow in a fractured porous medium and describe the coupling of the flow and mechanics problems, and illustrate the properties of this model through various numerical simulations.

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S. Day 1 17:30-18:00 (JST) = 09:30-10:00 (CET)

P. Victor A. Kovtunenko

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and Siberian Branch of the Russian Academy of Sciences, Russia

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T. Poroelastic Medium with Non-Penetrating Crack Driven by Hydraulic Fracture: Shape Derivative and Singular Solution

A. We suggest a new class of unilateral variational models describing a poroelastic medium, which contains a fluid-driven crack (fracture) subject to non-penetration between opposite crack faces. The poroelastic system is fully coupled and yields degenerate elliptic-parabolic equations. Well-posedness to the corresponding variational inequality is established based on the Rothe semi-discretization in time. The incremental solution is expressed by a saddle-point problem with respect to the unknown solid phase displacement, pore pressure, and contact force. Applying the Lagrange multiplier approach and Delfour-Zolésio theorem, formula for the shape derivative is derived, which implies the energy release rate under irreversible crack perturbations. In the plane isotropic setting, applying the Fourier analysis, a semi-analytic solution is derived as a power series in the sector of angle 2π with respect to distance to the crack-tip. A square-root singularity takes place, and no logarithmic terms occur in the asymptotic expansion. Integral formulas calculating stress intensity factors for the poroelastic medium with the non-penetrating crack are obtained.

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S. **Day 1 18:00-18:30 (JST) = 10:00-10:30 (CET)**

P. Sayahdin Alfat*,[1] and Masato Kimura[2]

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[2]*Kanazawa University, Japan*

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T. On Phase Field Method for the Modeling of Fluid-Driven Crack Propagation in Porous Medium

A. Based on an interesting and useful analogy in our phase field model for thermal fracturing [1], we can easily derive the phase field model for fluid-driven crack propagation in a porous medium. This study is divided into two parts. For the first part, we show the correspondence between thermoelasticity [2] and poroelasticity [3]. Since the poroelasticity theory and thermoelasticity have correspondence, we easily show the energy equality of the poroelasticity theory. In the second part, we introduce our phase field model with unilateral contact conditions for crack propagation due to water pressure by coupling with the Biot poroelasticity model and show its energy equality. For numerical experiments, we will show some examples related to cracks due to water pressure including cracks due to desiccation.

Keywords: Phase Field Model; Crack Propagation, Water Pressure, Energy Equality; Adaptive Finite Element Method; Porous Medium; Random Heterogeneous Materials.

References

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S. **Day 1 18:30-19:00 (JST) = 10:30-11:00 (CET)**

P. Keigo Wada

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T. Attempt on hydrodynamic model to describe tourists' movement

A. In Japan, the government says that the tourism contributes to the Japanese economy. However, the Japanese tourism GDP occupies only about 2% of the entire Japanese GDP and is very low compared to foreign countries. In order to raise tourism GDP, one of the important issues is the community development comfortable for both tourists and local residents, which is expected to increase the number of tourists visiting there. Focusing on this point, we now attempt on the construction of hydrodynamic model to describe tourists' movement. The visualization of group behavior based on the continuous change of density would be preferable rather than discrete data information. The prediction of group behavior is important to prevent terrible accidents due to the crowding of people attracted by some interest event in one spot. We will talk about the on-going work on the hydrodynamic model with some event information.

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S. **Day 1 19:00-19:30 (JST) = 11:00-11:30 (CET)**

P. Julius Fergy Rabago

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T. An ADMM numerical approach in shape optimization setting for geometric inverse problems

A. We propose the application of the alternating direction method of multipliers or ADMM in shape optimization framework for solving geometric inverse problems. We present some preliminary results illustrating the applicability and efficiency of the method by testing it to a shape detection problem with the focus on unknown inclusions having pronounced concavities and under noisy data. Examples are given in two and three dimensions.

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S. **Day 1 19:30-20:00 (JST) = 11:30-20:00 (CET)**

P. Pavel Strachota*, Michal Beneš, Pavel Eichler, Jan Hrdlička, Jakub Klinkovský, Miroslav Kolář, Tomáš Smejkal, Pavel Skopec, Jakub Solovský, and Alexandr Žák

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T. Experimental Validation of Multiphase Particle-in-Cell Simulations of Fluidization in a Bubbling Fluidized Bed Combustor

A. As part of our long-term aim at developing a complex CFD simulation tool for bubbling fluidized bed boilers operating in oxyfuel regime, we have performed several fluidization experiments to validate our numerical algorithm for multiphase flow simulation. The algorithm combines and extends the function of several components of OpenFOAM, using the Multiphase Particle In Cell method within a custom solver derived from coalChemistryFoam. In this contribution, we summarize the features of the numerical solver. Afterward, we focus on the design of the experiment, the methodology of data processing, and the obtained results.

The experimental device used is a model of the combustion chamber, the distributor, and the windbox. Its transparent walls are made of polymethyl methacrylate. In the experiments, three different materials (sand and two types of LECA suitable for use in fluidized bed combustion) were subjected to fluidization at different flow rates, using air at room temperature as the fluidization medium. The flow rates and the pressure drop at the fluidized bed were measured. Video recordings were taken and image postprocessing allowed to identify the level of the fluidized bed, its height and its shape as viewed from the front. Based on the sieve analysis, accurate particle size distributions were calculated and implemented in the model. Afterward, simulations were performed and model parameters (especially those of the particle stress model and the gas-solid drag model) were adjusted to obtain satisfactory agreement with the experiments in terms of the time-averaged pressure drop, bed height, and qualitative behavior.

In addition, bubble size statistics were obtained from sand simulations by taking advantage of the solids volume fraction field of the MP-PIC algorithm. The results were compared with data from literature and a surprisingly high degree of agreement has been found.

In summary, we present a unique combination of mathematical modeling, numerical simulation and experimental work aimed at creating a reliable computational model of a fluidized bed boiler.

Day 2

S. = Schedule · **P.** = Presenter · **T.** = Title · **A.** = Abstract

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S. Day 2 17:00-17:30 (JST) = 09:00-09:30 (CET)

P. Deepjyoti Goswami

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T. An Interior Penalty Discontinuous Galerkin Method for the Kelvin-Voigt Viscoelastic Fluid Model

A. An interior penalty discontinuous Galerkin method is formulated and analyzed for the Kelvin-Voight viscoelastic fluid model. Consistency of the scheme and well-posedness of the discrete discontinuous solutions have been established. Then optimal a priori error estimates for the velocity and the pressure in $L^\infty(L^2)$-norm are derived. Standard duality arguments and modified Sobolev-Stokes operator defined on appropriate broken Sobolev spaces are our main tools in our error analysis. Error estimates are shown to be uniform in time for sufficiently small data. Finally fully discrete case, based on the backward Euler scheme is analyzed and optimal fully discrete errors are derived. Our numerical experiments support our theoretical findings.

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S. Day 2 17:30-18:00 (JST) = 09:30-10:00 (CET)

P. Giulio G. Giusteri

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T. Tensorial transport equations for viscoelasticity and jamming

A. In this talk I will present a recent proposal for the description of materials that feature both fluid-like and solid-like behaviors, from mildly viscoelastic fluids to the extreme case of shear jamming of dense suspensions. In this line of research, the momentum balance equation is coupled with evolution equations for tensorial quantities that contribute in various ways to the total stress. Those equations describe evolutions on an infinite-dimensional manifold. As such, the existence of weak solutions in the classical sense of Banach or Hilbert spaces cannot be proved. I will discuss the notion of charted weak solutions that, together with a well-motivated design of the constitutive model, leads to a successful existence theory, meanwhile suggesting approximation strategies.

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S. Day 2 18:00-18:30 (JST) = 10:00-10:30 (CET)

P. Ryohei Seto

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T. Migration and jamming in pressure-driven dense suspension flows

A. Suspension is a mixture of solid particles and viscous liquid. It may flow as a fluid. At a high solid concentration (referred to as "dense"), suspensions exhibit a peculiar way of flowing. We can find numerous pieces of research to conduct rheological characterization of dense suspensions, which mostly report peculiar shear-rate dependences of viscosity, i.e., shear thinning and shear thickening. However, so far, no constative models satisfactorily can capture their flowing behavior; there seems to be a wide gap between the rheology and fluid mechanics of dense suspensions. I introduce our simulation approach to reproduce pressure-driven channel flows. We combine a Stokesian Dynamics-like particle simulation and coarse-grained computational fluid dynamics, which allows us to study nonuniform flows of dense suspensions. We demonstrate the pressure-difference dependence of channel flow rates for dense suspensions. (This work is a collaboration with Dr. Zhiyuan Zhao and Prof. Masao Doi.)

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S. Day 2 18:30-19:00 (JST) = 10:30-11:00 (CET)

P. Kohji Ohtsuka

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T. Shape optimization of singular points in boundary value problems

A. The talk will outline the paper [5] published by the American Mathematical Society. For more information and helpful figures, see the website `http://otlab.jp`. The paper describes a method for solving the problems **SoptS** that optimize sets of singular points such as boundaries, cracks and interfaces under cost functions in terms of energy, mean compliance and least-squares error. As the problems span many different fields, it is named **SoptS** for **S**hape **opt**imization of the sets of **S**ingular poits.

We consider SoptS for singular points of a weak solution in boundary value problems of partial differential equation/system defined in $\Omega \subset \mathbb{R}^d, d \geq 2$, including fracture mechanics [3], shape optimization of boundary [4] and interface,etc. Notice that the boundary $\partial\Omega$ is always considered as the set of singular points. Roughly speaking, singularity is a gap between weak and strong solutions. The shape sensitivity in SoptS is provided by the GJ-integral [3, 4] defined with an arbitrary domain ω as an extension of the J-integral [2, 6, 7] in fracture mechanics. The GJ-integral $J_\omega(u,\mu) := P_\omega(u,\mu) + R_\omega(u,\mu)$ is defined locally in $\omega \cap \Omega$ with a vector field $\mu \in W^{1,\infty}(D;\mathbb{R}^d), \overline{\Omega} \subset D$, is made without the boundary conditions. Here $P_\omega(u,\mu)$ is the surface integral on $\partial(\omega \cap \Omega)$ and the domain integral $R_\omega(u,\mu)$ in $\omega\cap\Omega$ is finite for the weak solution u. $J_\omega(u,\mu)$ is 0 if the solution u is regular inside $\omega \cap \Omega$, and $R_\Omega(u,\mu_\varphi)$ expresses energy shape sensitivity $\partial_\varphi \mathscr{E}(u)$ with respect to the mapping $[t \mapsto \varphi_t] \in C^1([0,t_0]; C^{0,1}(D;\mathbb{R}^d)), \varphi_0(x) = x, \mu_\varphi := d\varphi_t/dt|_{t=+0}$

$$\partial_\varphi \mathscr{E}(u) = \frac{d}{dt}\mathscr{E}(u(t); f, \Omega(t))|_{t=+0} = -R_\Omega(u,\mu_\varphi) - \int_{\partial\Omega} fu(\mu_\varphi \cdot n)\, ds, \tag{1}$$

$$u(t) \in \operatorname*{arg\,min}_{w \in V^\varphi(\Omega(t))} \int_{\Omega(t)} \{\widehat{W}(x, w, \nabla w) - fw\} dx \tag{2}$$

for the perturbation $\Omega(t) = \varphi_t(\Omega)$ and a given $f \in H^1(D;\mathbb{R}^m)$, $V^\varphi(\Omega(t)) := \{v \circ \varphi_t^{-1} : v \in V(\Omega)\}$ and $V(\Omega)$ the closed subset of $H^1(\Omega;\mathbb{R}^m), m \geq 1$ [5, Coro.2.9]. Here (2) is the partial differential equation when $m = 1$, otherwise system.

A generalization of the Hadamard variational formula [5, Th.3.1] is obtained using the adjoint variable method and various cost functions are rewritten using the GJ-integral [5, Sec.3]. The optimization is given by H1-gradient method [1, Th.7.6.2] using GJ-integral.

References

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S. Day 2 19:00-19:30 (JST) = 11:00-11:30 (CET)

P. Takashi Nakazawa

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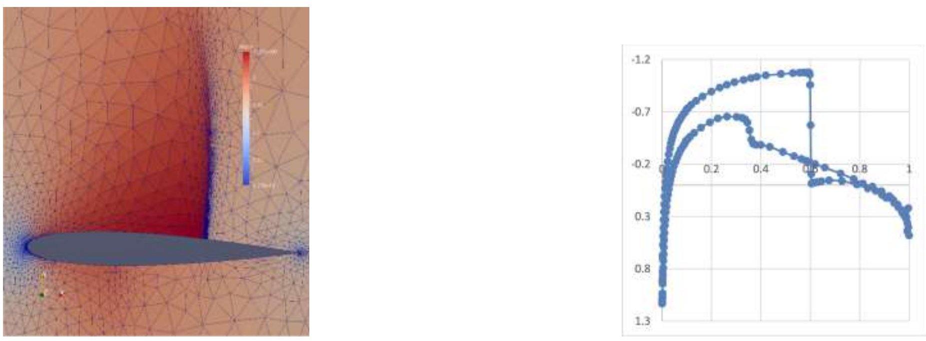
T. New numerical scheme for compressible flows

A. F. De Vuyst (HAL Id : cel-00842234, ver. 1) suggests a new mathematical model for compressible Euler equation as follows:

$$
\begin{aligned}
\frac{Da_\rho}{Dt} + \nabla \cdot \mathbf{u} = 0, &\quad a_\rho = \log(\rho),\\
\frac{1}{RT}\frac{D\mathbf{u}}{Dt} + \nabla a_\rho = 0,&\\
\frac{Da_\rho}{Dt} + \gamma\nabla \cdot \mathbf{u} = 0, &\quad a_\rho = \log(\rho).
\end{aligned}
$$

In general, compressible fluid is calculated by using finite volume method (FVM) or discontinuous Gelerkin method (DGM), to gurantee high numerical calculation accuracy. However, these discritizing sheme is needed high calculation cost. The above mathematical model is able to be used standard Gelerkin method with lower calculation cost than FVM and DGM. In this presentation, NACA0012 is adopted as a first trial domain, and calculation results are very similar to previous results.



(a) Mach number and FEM mesh. (b) Pressure distribution on NACA0012.

Figure 1: Numerical Results.

Keywords: Compressible Euler equation, Finite element method, NACA0012.