Advancements in Applied Mathematics:

A Virtual Workshop for Graduate Students in Applied Mathematics

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Advancements in Applied Mathematics:
A Virtual Workshop for Graduate Students in Applied Mathematics
for print use.
The website for the meeting can be found at:
`https://sites.google.com/view/iwgsam2024/home`

Contents

About

The Conference

This is a one-day international workshop for graduate students in applied mathematics.

The event has been organized with the purpose of introducing the ongoing research projects conducted by graduate students in applied mathematics. The primary goal of this event is to create a dynamic environment conducive to meaningful discussions and the exchange of innovative ideas. Throughout the workshop, active participation and engagement from all attendees is strongly encouraged. The organizers aim to provide a platform for graduate students to share their knowledge and insights, with the ultimate goal of inspiring collaboration and fostering further advancements in the field of Applied Mathematics.

The Organizers

Jerico Bacani	University of the Philippines Baguio
Masato Kimura	Kanazawa University, Japan
Saraleen Mae Manongsong	University of the Philippines Baguio
Renier Mendoza	University of the Philippines Diliman
Hirofumi Notsu	Kanazawa University, Japan
Gilbert Peralta	University of the Philippines Baguio
Julius Fergy Rabago	Kanazawa University, Japan
John Sebastian Simon	RICAM, Austria

Registration

Please register through the link: `https://us06web.zoom.us/meeting/register/`.

Timetable

Japan Standard Time

PST, CET Conversion[1]

15:00 - 15:05	**Welcome Remarks**	
Session 1	**February 14, 2024**	
15:05 - 15:17	**Elsa Bernholm** Japan	Numerical validation of a Stochastic Delay Differential Equation model for the human body balance
15:17 - 15:29	**Junius Wilhelm Bueno** Philippines	Navier–Stokes–Voigt system with forces of low spatial Regularity
15:29 - 15:41	**Yosuke Mizuno** Japan	Universal approximation of deep learning ODENet defined by solutions of ordinary differential equations
15:41 - 15:53	**Ghulam Abrar** Japan	Exploring the potential of Physics-Informed Neural Networks in solving nonlinear PDEs: a case study with the p-Poisson equation
15:53 - 16:00	**Break**	*
Session 2	**February 14, 2024**	
16:00 - 16:12	**Joseph Ludwin DC. Marigmen** Philippines	Analysis on the meteorological factors affecting trend of weekly dengue incidence in Baguio City from 2010 to 2022
16:12 - 16:24	**Keisuke Mochizuki** Japan	Finite element simulation of fracture phase field models
16:24 - 16:36	**Andhee Jacobe** Philippines	Deterministic and stochastic analyses of an HIV/AIDS transmission model with two infectious compartments and standard incidence rate
16:36 - 16:48	**Masato Taketomi** Japan	Phase field model for fracture in plate bending problem
16:48 - 16:55	**Break**	*

[1] Time minus one (1) hour for Philippine Standard Time and minus eight (8) hours for Central European Time.

Session 3	February 14, 2024	
16:55 - 17:07	**Cielo Denise Gumaru** Philippines	Finite element approximation of the Cahn-Hilliard-Oberbeck-Boussinesq system
17:07 - 17:19	**Md Mamun Miah** Japan	Numerical study on crack path by fracture phase field model with unilateral contact condition
17:19 - 17:31	**Rolly Czar Joseph Castillo** Philippines	On the smoothing of 1D and 2D data in Sobolev space
17:31 - 17:43	**Ryuhei Wakida** Japan	Simulation of a fault-type earthquake by phase field crack growth model
17:43 - 17:50	**Break**	*
Session 4	**February 14, 2024**	
17:50 - 18:02	**Yuma Nakamura** Japan	On the memory of the twin vortex computer for an optimized cylinder
18:02 - 18:14	**Sadique Rehman** Japan	A Lagrange-Galerkin finite element method for a naturally convected viscoelastic Maxwell fluid model
18:14 - 18:26	**Stanislav Mosný** Sweden	Data assimilation in the theory of non-Newtonian fluids
18:26 - 18:35	**Closing Remarks**	

Presentation Format

Ten (10) minutes are allotted to each speaker, followed by two (2) minutes for a question-and-answer period.

Abstracts and Presenters (Alphabetical)

Exploring the potential of physics-informed neural networks in solving nonlinear PDEs: a case study with the p-Poisson equation

Ghulam Abrar[a]***, Hirofumi Notsu***[b]***, and Julius Fergy Tiongson Rabago***[b]

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In this study, we explore the field of nonlinear partial differential equations (PDEs), particularly focusing on the p-Poisson equation, a complex variant of the well-known Poisson equation. Our approach utilizes the advantage of the application of Physics-Informed Neural Networks (PINNs), exploring their effectiveness in handling the non-linear challenges posed by the p-Laplacian operator.

Our study examines several neural network architectures and their effects on various parameters, including learning rates, layer numbers, neuron count per layer, and initialization methods – which influence the accuracy of our solutions. We specifically address the hurdles in solving the equation for diverse values of the nonlinearity parameter, p. This covers an analysis of the architecture of the neural network and its function in providing accurate solution approximations.

Our investigation found that although there is not a universal network architecture that works for all situations, certain PINN configurations are particularly adept at solving the p-Poisson equation efficiently. This presentation will offer a glimpse into our findings, underscoring the adaptability and promise of neural networks in tackling complex nonlinear PDEs. This overview aims to contribute to the ongoing conversation in computational mathematics and the expanding role of machine-learning techniques in this field.

Numerical validation of a Stochastic Delay Differential Equation model for the human body balance

Elsa Bernholm[a]***, Thomas De Jong***[b]***, and Masato Kimura***[b]

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This report focus to find a model to describe healthy peoples balance. An anzats of a Stochastic Delay differential equation is used and investigated. The Stochastic Delay differential Equation (SDDE) is described as follows

$$M\ddot{x} = -Ax(t-\tau_x) - B\dot{x}(t-\tau_v) + f(t),$$

where $x(t)$ is the position depending of time t, τ_x and τ_v is time delays. Moreover M is a diagonal matrix with the mass m of the body, A and B is diagonal matrices. While $f(t)$ is the stochastic term. Comparison with samples of data is made to determine the models reliability.

Navier–Stokes–Voigt system with forces of low spatial regularity

Junius Wilhelm Bueno

Department of Mathematics and Computer Science, University of the Philippines Baguio, Philippines

The Navier–Stokes–Voigt system models motion of viscous incompressible Newtonian fluid under an abrupt force. In this presentation, we study this system in dimension two under forces in the Sobolev space of negative orders. We prove the existence and uniqueness of solutions for the associated Stokes equation. Measure-valued sources are covered in this framework.

On the smoothing of 1D and 2D data in Sobolev space

Rolly Czar Joseph Castillo

Institute of Mathematics, University of the Philippines Diliman, Philippines

In this talk, we pose data smoothing as minimization problems in the Sobolev space. For one-dimensional data, our data smoothing approach generalizes the Whittaker-Henderson method by projecting the interpolation of noisy data points to a finite-dimensional subspace of the Sobolev space $H^m(\Omega)$. The resulting smoothed data is represented by a polynomial whose degree can be calibrated or optimized by the user. For two-dimensional data, we solve a half-quadratic minimization problem whose optimality condition is an elliptic partial differential equation (PDE). We then solve this PDE using the finite element method.

We present applications of our approach on extracting trends in infectious disease data and in image de-noising. The resulting polynomial for smoothed 1D data can be used in generating short-term forecast of infectious diseases. Meanwhile, the image de-noising approach can be combined with other machine learning techniques such as the K-means in edge detection in images.

References

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[2] Mendoza, R. and Recio, K.R. (2023). A Hybrid of Half-Quadratic Minimization Algorithm and Method of K-means for Edge Detection of Texts. Preprint.

Deterministic and Stochastic Analyses of an HIV/AIDS Transmission Model with Two Infectious Compartments and Standard Incidence Rate

Andhee Jacobe

Department of Mathematics and Computer Science, University of the Philippines Baguio, Philippines

This study investigated the behavior and biological feasibility of a compartmental model of HIV/AIDS transmission. A deterministic system of ordinary differential equations (ODEs) was first used as the model's mathematical representation and was established to be well-posed. Moreover, there were two equilibrium points — one disease-free and one endemic. The next-generation approach was used to derive the reproduction number $\mathcal{R}_0$, which is found to be drastically affected by the introduced interaction between susceptible and AIDS-afflicted individuals. Exploration of the global stability of the equilibria through numerical simulations was also considered.

A stochastic extension of the system of ODEs was also investigated. Well-posedness of the system of stochastic differential equations was assured provided that the diffusion part was adjusted when there are near-zero values. Mean stability, dispersion patterns, and extinction probabilities were explored through numerical simulations.

Finite Element Approximation of the Cahn-Hilliard-Oberbeck-Boussinesq System

Cielo Denise Gumaru

Department of Mathematics and Computer Science, University of the Philippines Baguio, Philippines

We consider the coupling of the two-dimensional Cahn-Hilliard and Oberbeck-Boussinesq systems which yields a phase-field model for non-isothermal, incompressible and viscous flows. A derivation of the fully discretized finite element approximation of this model as well as the assembly of the resulting finite element matrices are presented for analysis. Numerical experiments are conducted to illustrate the features of the proposed model.

Analysis on the Meteorological Factors Affecting Trend of Weekly Dengue Incidence in Baguio City from 2010 to 2022

Joseph Ludwin DC. Marigmen

Department of Mathematics and Computer Science, University of the Philippines Baguio, Philippines

Reports of increasing dengue incidence in areas are observed due to meteorological factors on a certain period or season in an area. Utilizing relative humidity, temperature, and precipitation, this study analyzes the effects of meteorological factors on the trend of weekly dengue incidence in Baguio City from 2010 to 2022. In this study, polynomial regression is utilized in observing the trend of dengue incidence throughout the 52-week period, while multiple linear regression is conducted in associating these factors. As a result, dengue incidence are relatively high between week 23 and week 38, or between the months of July and September. Further analyzing the trend, relative humidity and maximum temperature provide significantly strong contribution as factors to dengue incidence. These results can assist the local government in controlling and preventing possible influx of incidence in the city.

Numerical study on crack path by fracture phase field model with unilateral contact condition

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Fracture problems in modern science and technology are prevalent and severe issues nowadays. For the numerical simulation of crack propagation, the fracture phase field model (F-PFM) is commonly used in fracture mechanics. We examine the numerical simulation for crack propagation due to the various scenarios, such as Mode I crack propagation, mixed Mode crack propagation, and compression in the domain with inclined and horizontal cracks, by using the finite element method (FEM) for the F-PFM. In every instance, we observed the realistic fracture propagation feature. For the case of nonrealistic, we mentioned the F-PFM with unilateral contact condition. Here we also proposed the F-PFM with unilateral contact condition to avoid nonrealistic cases of crack propagation. By means of the original F-PFM and the F-PFM with unilateral contact condition, we noticed that Mode I and crack propagation due to compressing in the domain with horizontal crack both yield realistic results. The F-PFM with unilateral contact condition gave us a realistic crack path for the mixed mode, whereas the original F-PFM gave us a branching crack path, which is not possible. For crack propagation due to compressing in the domain with inclined crack, the original F-PFM is not realistic but the crack profile is realistic in the F-PFM with unilateral contact condition. Thus, we may conclude that in comparison to the original F-PFM, crack propagation resulting from unilateral contact circumstances produces more realistic findings. In some circumstances, we also go over the driving force profile and surface energy of the original and unilateral contact condition. Here, we used FreeFem++ software for numerical simulation and symmetric mesh generation with adaptation.

Universal approximation of deep learning ODENet defined by solutions of ordinary differential equations

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We consider the following initial value problem of a system of ODEs:

$$\begin{cases} x'(t) = f(x(t), t) \quad (0 \leq t \leq T), \\ x(0) = \xi \in D, \end{cases} \tag{1}$$

which is called a neural ordinary differential equation (NODE). We call the function $D \ni \xi \mapsto x(T) \in \mathbb{R}^N$ an ODENet with a vector field f. We restrict the function f to the following form:

$$f(x, t) = \alpha(t) \odot \boldsymbol{\sigma}(\beta(t)x + \gamma(t)), \tag{2}$$

where $\alpha : [0, T] \to \mathbb{R}^N$ is a weight vector, $\beta : [0, T] \to \mathbb{R}^{N \times N}$ is a weight matrix, and $\gamma : [0, T] \to \mathbb{R}^N$ is a bias vector. The operator $\odot$ denotes the Hadamard product (element-wise product) of two vectors. The function $\boldsymbol{\sigma} : \mathbb{R}^N \to \mathbb{R}^N$ is defined by

$$\boldsymbol{\sigma}(x) := (\sigma(x_1), \sigma(x_2), \cdots, \sigma(x_N))^\top, \tag{3}$$

where $\sigma : \mathbb{R} \to \mathbb{R}$ is called an activation function.

For the ordinary differential equation (1), we call $S_f(t)\xi := x(t) \quad (0 \leq t \leq T)$ a "flow" associated with the vector field f and call $S_f(T) : \xi \mapsto x(T)$ ODENet with respect to f. Let $\sigma : \mathbb{R} \to \mathbb{R}$ and let D be a bounded closed subset of $\mathbb{R}^N$. Additionally, consider the set

$$S := \left\{ G : D \to \mathbb{R} \,\middle|\, G(\xi) = \sum_{l=1}^{L} \alpha_l \sigma(\beta_l \cdot \xi + \gamma_l), L \in \mathbb{N}, \alpha_l, \gamma_l \in \mathbb{R}, \beta_l \in \mathbb{R}^N \right\}.$$

Suppose that S is dense in $C^0(D)$, i.e., given $F \in C^0(D)$ and $\epsilon > 0$, there exists a function $G \in S$ such that

$$|G(\xi) - F(\xi)| < \epsilon$$

for any $\xi \in D$. Then, we say that σ has a universal approximation property (UAP) on D.
In this presentation, we establish the conditions for UAP in the sup-norm for the ODENet with respect to any non-polynomial continuous activation function that satisfies the Lipschitz condition. Here, the layer width is N. Specifically, we prove that if σ has UAP, then the ODENet with respect to any function satisfying the Lipschitz condition can be approximated by the ODENet with respect to a function of type (2). In other words, let $D \subset \mathbb{R}^N$ be a bounded closed subset. If $\sigma \in C^0(\mathbb{R})$ satisfies UAP and the Lipschitz condition with respect to x and $f \in C^0(\mathbb{R}^N \times [0, T]; \mathbb{R}^N)$ satisfies the Lipschitz condition with respect to x, then given $\epsilon > 0$, there exist $\alpha, \gamma \in C^0([0, T]; \mathbb{R}^N)$ and $\beta \in C^0([0, T]; \mathbb{R}^{N \times N})$ such that

$$\|S_f(t) - S_h(t)\|_{C^0(D;\mathbb{R}^N)} < \epsilon \quad (0 \leq t \leq T) \tag{4}$$

where $h(x, t) = \alpha(t) \odot \boldsymbol{\sigma}(\beta(t)x + \gamma(t))$.

Data assimilation in the theory of non-Newtonian fluids

Stanislav Mosný

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The Ladyzhenskaya model is a generalization of the Navier-Stokes model, which can be used to describe the behavior of non-Newtonian fluids. In this talk, we will first discuss the well-posedness of this problem. The main objective is to study data assimilation for the Ladyzhenskaya model, which aims to find an approximation of the reference solution when the initial data is unknown. Instead of having initial data, we possess measurements of the reference solution. We will demonstrate that the data assimilation algorithm constructs an approximation that converges to the reference solution over time. The critical aspect of this study involves analyzing the long-term behavior of the reference solution.

Finite element simulation of fracture phase field models

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Introduction. This paper aims to simulate crack propagation phenomena under various conditions. First, we derive a model for crack growth. Let Ω be a two-dimensional bounded domain and Σ be a crack. Elastic body containing crack is described as the following boundary value problem of partial differential equations:

$$\begin{cases} -\text{div}\sigma[u] = f(x,t) & \text{in} \quad \Omega\backslash\Sigma \times [0,T] \\ u = g(x,t) & \text{on} \quad \Gamma_D \times [0,T] \\ \sigma[u]\nu = q(x,t) & \text{on} \quad \Gamma_N \times [0,T] \\ \sigma[u]\nu = 0 & \text{on} \quad \Sigma^{\pm} \times [0,T] \end{cases} \tag{1}$$

Phase field model. The partial differential equations of (1) does not have a term that describes the crack state and cannot accurately describe the crack growth phenomenon. Therefore, we apply phase file function $z(x,t)$ to represent crack state [2,3,4]. We assume that $0 \leq z(x,t) \leq 1$ and $z(x,t) \approx 1$ around Σ and that $z(x,t) \approx 0$ for the other region. By introducing the phase field function $z(x,t)$ and spatial regularization parameter, the elastic energy $E1$, interface energy $E2$, and total energy E can be described as (2).

$$\begin{cases} E_1(t,u,z) := \dfrac{1}{2}\displaystyle\int_\Omega (1-z)^2\sigma[u] : e[u]\, dx \\ E_2(z) := \dfrac{1}{2}\displaystyle\int_\Omega G_c(\epsilon|\nabla z|^2 + \dfrac{1}{\epsilon}z^2)\, dx \\ E(t,u,z) := E_1(t,u,z) + E_2(z) \end{cases} \tag{2}$$

Where G_c is a constant that expresses the fragility of an object. We define the elastic energy density as $w[u] := \frac{1}{2}\sigma[u] : e[u]$. By setting $f = 0, q = 0$ and computing the gradient flow in (2), the following phase field equations are obtained.

$$
\begin{cases}
-\text{div}((1-z)^2\sigma[u]) = 0 & \text{in} \quad \Omega\times[0,T] \\
\dfrac{\partial z}{\partial t} = \Big(\epsilon\text{div}\,(G_c\nabla z) & \\
\qquad -\dfrac{G_c}{\epsilon}z + 2(1-z)w[u]\Big)_+ & \text{in} \quad \Omega\times[0,T] \\
u = g(x,t) & \text{on} \quad \Gamma_D\times[0,T] \\
\sigma[u]\nu = 0 & \text{on} \quad \Gamma_N\times[0,T] \\
\dfrac{\partial z}{\partial \nu} = 0 & \text{on} \quad \Gamma\backslash\Gamma_N\times[0,T] \\
z = 0 & \text{on} \quad \Gamma_N\times[0,T] \\
z|_{t=0} = z^0 & \text{in} \quad \Omega
\end{cases}
\tag{3}
$$

Compression conditions. We perform a crack growth simulation under compression conditions. We introduce the function $\chi\left(x\right)$ and decompose the elastic energy density into three components expansion $W_+[u]$, compression $W_-[u]$, and shear $W_0[u]$ [5,6]. We assume that $\chi(x)=1$ when div$u\geq 0$ and that $\chi(x)=0$ when div$u<0$. We redefine elastic energy to avoid phase field effects in the compressive component as

$$
E_1(t,u,z) = \int_\Omega (1-z)^2\left(w[u]_+ w[u]_0\right) + w_-[u]\; dx \tag{4}
$$

Traveling wave solution. We consider an finite strip domain $\Omega_H := \mathbb{R}\times(-H,H)$ and consider a traveling wave solution of the phase field model [7]. An initial crack is made along the x_1 axis and a constant force g_0 is applied from the top (x_1,H) and bottom $(-x_1,H)$ of the domain.

$$
\begin{cases}
-\text{div}((1-z)^2\sigma[u]) = 0 & \text{in} \quad \Omega_H \\
\alpha_2\dfrac{\partial z}{\partial t} = \left(\epsilon\text{div}(G_c\nabla z) - \dfrac{G_c}{\epsilon}z + (1-z)\sigma[u]:e[u]\right)_+ & \text{in} \quad \Omega_H \\
u = \begin{pmatrix} 0 \\ \pm g_0 \end{pmatrix} & \text{on} \quad \Gamma_D \\
\dfrac{\partial z}{\partial x_2} = 0 & \text{on} \quad \Gamma_D
\end{cases}
\tag{5}
$$

We suppose that there exists a traveling wave solution with a velocity $V>0$ in the direction of positive x_1. Then there exists $\tilde{u}:\Omega_H\to\mathbb{R}$, $\tilde{z}:\Omega_H\to(0,1)$ and $V>0$ such that $u\left(x_1,x_2,t\right)=\tilde{u}\left(x_1-Vt,x_2\right)=\tilde{u}\left(\xi\right)$, $z\left(x_1,x_2,t\right)=\tilde{z}\left(x_1-Vt,x_2\right)=\tilde{z}\left(\xi\right)$, where we set a moving coordinate $\xi:=(x_1-Vt,x_2)$. Then, since $\frac{\partial z(x,t)}{\partial t}=-V\frac{\partial \tilde{z(\xi)}}{\partial x_1}$, $(\tilde{u},\tilde{z},V)$ is a solution of the following system:

$$
\begin{cases}
-\mathrm{div}((1-\tilde{z})^2\sigma[\tilde{u}]) = 0 & \text{in} \quad \Omega_H \\
-\alpha_2 V \dfrac{\partial \tilde{z}}{\partial x_1} = \epsilon \mathrm{div}(G_c \nabla \tilde{z}) - \dfrac{G_c}{\epsilon} z + (1-\tilde{z})\sigma[\tilde{u}] : e[\tilde{u}] & \text{in} \quad \Omega_H \\
\tilde{u} = \begin{pmatrix} 0 \\ \pm g_0 \end{pmatrix} & \text{on} \quad \Gamma_D \\
\dfrac{\partial \tilde{z}}{\partial x_2} = 0 & \text{on} \quad \Gamma_D
\end{cases}
\tag{6}
$$

We investigate the relationship between the crack growth velocity V and each parameter.

References

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On the memory of the twin vortex computer for an optimized cylinder

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In recent years, forecasting certain phenomena based on given robust data has been of interest among mathematicians and allied scientists. Our interest lies in the study of a method known as *Physical Reservoir Computing* which is derived from the framework of recurrent neural network. In particular, we consider the dynamics induced by the flow past a cylinder which was studied in [1]. In the said article the authors studied the effect of varying the so-called Reynolds number to the information processing capacity. They showed that the case where the twin-vortex — generated due to the periodically stable flow — appears, the physical reservoir computer gives a meaningful information processing and reaches peak capability when the Reynolds number is at the bifurcation point of stability. In this talk, we present an on going study on the effect of varying the shape of the obstacle in the channel by using shape optimization. The optimization problem is formulated so as to either maximize or minimize the L^2-norm of the curl of the velocity field based on the result of [1] indicating that the size of the vortex affects the computational capability of the system. With the use of traction method based on gradient descent of the vorticity functional, we numerically solve the shape optimization problem. The performance of the physical reservoir computer is then evaluated on the shapes generated from the optimization problems. We then present our early results which shows that the shapes generated by the minimization problem give a better performance compared to the maximized one. This is a joint work with John Sebastian H. Simon, Tomoyuki Kubota, Kohei Nakajima, and Hirofumi Notsu.

References

[1] K. Goto, K. Nakajima and H. Notsu. Twin vortex computer in fluid flow. *New Journal of Physics*, 23: 063051, 2021.

A Lagrange-Galerkin finite element method for a naturally convected viscoelastic Maxwell fluid model

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The main purpose of this work is to develop the stable finite element scheme for the incompressible viscoelastic Maxwell fluid model coupled with the natural convection phenomenon. The upper-convected time derivative is discretized through the so-called Lie derivative. The P1 finite element space is chosen with the pressure-stabilization. Firstly, to validate our scheme, we compute experimental orders of convergence with suitable norms and observe that the velocity, pressure, stress, and temperature are first-order convergent in time and space. Secondly, for a better understanding of the flow pattern, we consider the square cavity having a hot left boundary and a cold right boundary. The eye of the recirculation zone is toward the upstream region when the Weissenberg number increases. We compare the results obtained by our scheme with the ones given in the existing literature for purely naturally convected flow through the values of the Nusselt number. They are in good agreement with the literature. Finally, we show the elastic effect in the cavity flow problem for a naturally convected viscoelastic Maxwell fluid model.

Phase field model for fracture in plate bending problem

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The phase field model is attracting attention as model enable crack growth simulation. In the phase-field model, crack is expressed by a smooth damage variable $z(x,t) \in$[0, 1], and we can various crack growth simulation. Although the extension to crack growth by bending plates has not yet been completed.

The final goal is using the finite element method to calculate the problem of crack growth caused by bending a plate. It is necessary to consider the variable bending equation plate. It is known that the equation with constant is the same as the constant biharmonic equation. Therefore, we thought that the equation with variable coefficients is also related to biharmonic equations. Researching constant biharmonic equations has been advanced. But there are few literatures on biharmonic equations with variable coefficients. The main theme of this research is to transform variable biharmonic equations with variable coefficients into formulas that can be calculated numerically, and to develop crack growth problems.

Variable biharmonic equation. $\Omega \subset \mathbb{R}^n$ is Bounded Lipschitz domain. Γ is boundary of Ω. Γ_D is Dirichlet boundary and $\Gamma_N(=\Gamma\backslash\Gamma_D)$ is Neumann boundary. $d(x)$ is smooth function. The boundary conditions are as follows.

$$\begin{cases} \Delta(d(x)\Delta u(x)) = f(x) & \text{in } \Omega \\ u = 0, \frac{\partial u}{\partial \nu} = g \ \text{ on } \ \Gamma_D \\ \frac{\partial u}{\partial \nu} = 0, \frac{\partial}{\partial \nu}(d(x)\Delta u) = 0 & \text{on } \ \Gamma_N \end{cases}$$

Using $v := -d(x)\Delta u$ can transform biharmonic equation.

$$\begin{cases} -\Delta u(x) = \frac{v}{d(x)} & \text{in } \Omega \\ -\Delta v = f & \text{in } \Omega \\ u = 0, \frac{\partial u}{\partial \nu} = g \ \text{ on } \ \Gamma_D \\ \frac{\partial u}{\partial \nu} = 0, \frac{\partial}{\partial \nu} v = 0 & \text{on } \ \Gamma_N \end{cases} \tag{1}$$

We use Mixed method proposed by Ciarlet-Raviart to calculate variable biharmonic equation.The test function for (u,v) is (φ,ψ).In this method,I consider each test function in reserve.We translated (1) into following weak equation.

$$\begin{cases} (u,v) \in H^1_{\Gamma_D}(\Omega) \times H^1(\Omega) \\ \displaystyle\int_\Omega \nabla u \nabla \psi dx - \int_\Omega \frac{v}{d} \psi dx = \int_\Gamma g_1 \psi dS \ \ (\forall \psi \in H^1(\Omega)) \\ \displaystyle\int_\Omega \nabla v \nabla \varphi dx = \int_\Omega f \varphi dx \ \ (\forall \varphi \in H^1_{\Gamma_D}) \end{cases} \tag{2}$$

Phase-field model of failure in plate bending problems. It is necessary to consider the plate bending equation with variable coefficients.We consider a two dimensional region.$\Omega \subset \mathbb{R}^2$ is Bounded Lipschitz domain.The plate($\Omega \times (0, b)$) is thick $b(> 0)$.$u(x) \in \mathbb{R}(x \in \Omega)$ is displacement and $f(x) \in \mathbb{R}(x \in \Omega)$ is vertical force. We introduced Young's modulus($E_Y(x) > 0$) and Poisson's ratio($\nu_p \in (-1, \frac{1}{2})$).

Bending plate equation. Consider the plate bending equation from the moment tensor($M(x) \in \mathbb{R}^{2\times2}_{\text{sym}}$)and force balance equation. Let $D^2u(x)$ be Hessian of u and I be identity matrix. We defined$d(x)$,$K[u](x)$ and $M(x)$ as follows.

$$d(x) := \frac{E_Y(x)}{1-\nu_p}\frac{b^3}{12}$$

$$K[u](x) := \nu_p \Delta u(x) I + (1-\nu_p) D^2 u$$

$$M(x) := -d(x)K[u]$$

The boundary conditions are as follows.

$$\begin{cases} \text{div}(\text{div } M) + f = 0 & \text{in } \Omega \\ u = g_0, \dfrac{\partial u}{\partial \nu} = g_1 & \text{on } \Gamma \end{cases} \tag{3}$$

The weak form of (3) is described below.

$$\begin{cases} \text{Given} \quad g = g_0, \ \dfrac{\partial g}{\partial \nu} = g_1, \ \text{ on } \Gamma, \\ u \in H^2(\Omega), u - g \in H^2_0(\Omega) \\ \displaystyle\int_\Omega d(x)\{(1-\nu_p)D^2u : D^2\varphi + \nu_p(\Delta u)(\Delta\varphi)\}dx \\ = \displaystyle\int_\Omega f\varphi dx \quad (\varphi \in H^2_0(\Omega)) \end{cases} \tag{4}$$

It is difficult to calculate of (4) as Ciarlet-Raviart mixed method.Constant bending equation match constant biharmonic equation.So we regarded (4) as variable biharmonic equation by $D^2u : D^2\varphi$=$(\Delta u)(\Delta\varphi)$.

Phase field model. We used Phase Field Model for the model of crack growth.Kimura-Takaishi combined with the time evolution of the phase field and the equation satisfied by the displacement of the bending plate equation using the energy expression of Francfort and Marigo.They proposed the following energy expression.$z(x, t) \in$[0, 1] is smooth damage variable.It means that cracks are formed where $z(x)$ is close to 1,and not cracked where z is close to 0.Near Phase Field parameter,Young's modulus is approximated to 0.So Young's modulus and d(x) change as follows.E^*_Y is Young's modulus when there are no cracks.

$$\begin{cases} d(x) = d(z) = \frac{((1-z(x))^2)E^*_Y}{1-\nu_p}\frac{b^3}{12} \end{cases}$$

I define other constants as follows.

$$\begin{cases} d_0 = \frac{E_Y^*}{1-\nu_p}\frac{b^3}{12} \\ G_c > 0 : \text{Fracture toughness value} \\ \epsilon > 0 : \text{Spatial regularity parameter} \\ 0 << \alpha << 1 : \text{Time constant} \end{cases}$$

Using Phase Field model,we constructed this model.

$$\begin{cases} \Delta(d(z)\Delta u) = f \ \text{ in } \ \Omega \ \times [0,T] \\ \alpha\dfrac{\partial u}{\partial t} = (G_c(\epsilon\Delta z - \dfrac{z}{\epsilon}) + d_0(1-z)|\Delta u|^2) \ \text{ in } \Omega \\ u = g_0, \dfrac{\partial u}{\partial \nu} = g_1 \ \text{ on } \Gamma_D \times [0,T] \\ \dfrac{\partial u}{\partial \nu} = 0, \dfrac{\partial}{\partial \nu}(d(z)\Delta u) = 0 \ \text{ on } \Gamma_N \times [0,T] \\ \dfrac{\partial z}{\partial \nu} = 0 \ \text{ on } \Gamma \times [0,T] \\ z|_{t=0} = z^0 \ \text{ in } \Omega \end{cases} \tag{5}$$

Result. We extended the variable biharmonic equation to Ciarlet-Raviart equation.We did error analysis using the finite element method and showed that the solution converged. We also tried to extend the variable bending plate equation to the Ciarlet-Raviart equation, but it did not work. Therefore, variable biharmonic equation was used as the variable banding plate equation and it was extended to crack growth phase field model.We simulate crack growth phase field model.

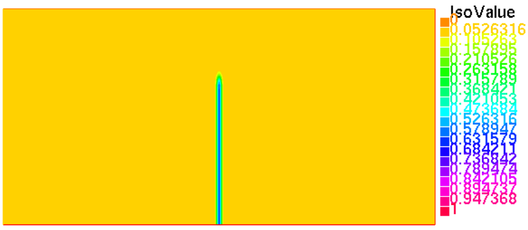

Figure 1: t=0

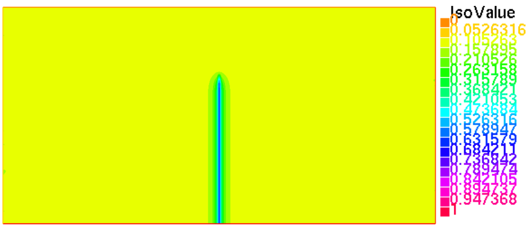

Figure 2: t=14

We expected growing crack in straight. The numerical Crack is not successful. Future work will be to find a method that can calculate variable bending plate equation and extend it to phase field model.

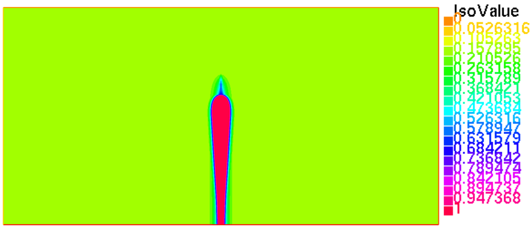

Figure 3: t=28

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Simulation of a fault-type earthquake by phase field crack growth model

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In Japan, an earthquake often occurs and it causes heavy damage sometimes. An earthquake can be roughly divided into 2 types, fault-type and trench-type. The purpose of our research is reproducing an fault-type earthquake numerically. Incidentally, Noto Peninsula Earthquake on New Years Day is a fault-type earthquake.

This study is based on phase field crack growth model, and we expand the model to wave equation. Thus, we can reproduce a fault and an earthquake wave simply. Note that this model does not contain friction conditions. The crack growth model is as below.

$$
\begin{cases}
\dfrac{\partial^2 u}{\partial t^2} &= \mathrm{div}((1-z)^2\sigma[u]) & x \in \Omega \\
\alpha\dfrac{\partial z}{\partial t} &= (\epsilon \mathrm{div}(\gamma(x)\nabla z) - \dfrac{\gamma(x)}{\epsilon}z + \sigma[u] : e[u](1-z))_+ & x \in \Omega \\
u(x,t) &= g(x,t) & \text{on} \quad \Gamma_D \\
\dfrac{\partial u}{\partial n} &= 0 & \text{on} \quad \Gamma_N \\
\dfrac{\partial z}{\partial n} &= 0 & \text{on} \quad \Gamma \\
 & \text{and I.C.} &
\end{cases}
$$

u and z are displacement and phase field function for damage respectively.

For numerical simulation, we use “FreeFem++” that a software focus on solving partial differential equations using the finite element method.

In presentation, I will display some numerical results. However, consideration remains unsatisfactory.