Young Researchers Workshop on Applied Mathematics

The Kanazawa Theatre, Kanazawa City, Japan, July 24 (Friday), 2026

Schedule of Talks

Time	**Speaker / Affiliation**
09:25–09:30	*Opening Remarks*, Koya Sakakibara, Kanazawa University, Japan
09:30–10:00	Michal BENEŠ, Czech Technical University in Prague, Czech Republic
10:00–10:15	Denisa HANUŠKOVÁ, Czech Technical University in Prague, Czech Republic
10:15–10:30	Kharisma Surya PUTRI, Kanazawa University, Japan
10:30–10:45	Kazuki OOKOMA, Meiji University, Japan
10:45–11:00	Coffee Break
11:00–11:15	Miroslav KNOBLOCH, Czech Technical University in Prague, Czech Republic
11:15–11:30	Kouta ARAKI, Nihon University, Japan
11:30–11:45	Shuhei SASAJIMA, Kanazawa University, Japan
11:45–12:00	Haruki TAKEMURA, The University of Tokyo, Japan
12:00–14:00	Lunch Break
14:00–14:30	Petr PAUS, Czech Technical University in Prague, Czech Republic
14:30–14:45	Florian GRUEN, Kyoto University, Japan
14:45–15:00	Shunsuke KASAO, Kanazawa University, Japan
15:00–15:15	Ondřej SAKALA, Czech Technical University in Prague, Czech Republic
15:15–15:30	Coffee Break
15:30–15:45	Yuma NAKAMURA, Kanazawa University, Japan
15:45–16:00	Tomáš NGUYEN, Czech Technical University in Prague, Czech Republic
16:00–16:15	Dalim HAQUE, Kanazawa University, Japan
16:15–16:30	Kryštof Deutschar BLAŽEK, Czech Technical University in Prague, Czech Republic
16:30–16:45	Hinata MOROOKA, Kanazawa University, Japan
16:45–17:00	Coffee Break
17:00–17:15	Sahat Pandapotan NAINGGOLAN, Kanazawa University, Japan
17:15–17:30	Subham PRASAD, Indian Institute of Technology Mandi, India
17:30–17:45	Oussama OUNISSI, Kanazawa University, Japan
17:45–18:00	Melvin Aballa OJWOK, Karlstad University, Sweden and University of L'Aquila, Italy
18:00–18:05	*Closing Remarks*, Masato Kimura, Kanazawa University, Japan

Instructions for Speakers

- **30-minute talk:** 25 min presentation + 5 min moderator's introduction and Q&A.
- **15-minute talk:** 12 min presentation + 3 min moderator's introduction and Q&A.

Please keep your presentation within the allotted time.

Schedule and Abstracts

Time	Title of Talk / Speaker / Affiliation / Abstract
09:25–09:30	*Opening Remarks* Koya Sakakibara, Kanazawa University, Japan
	Session 1
09:30–10:00	*Diffuse-interface approach to planar forced curvature flow with constraints* Michal Beneš Czech Technical University in Prague, Czech Republic In the contribution we discuss numerical solution of the planar curvature flow with the area constraint and external force, and its applications. The underlying motion law is captured by the phase-field method using the non-local Allen–Cahn equation, and compared to the parametric method in selected cases. We summarize the theoretical background of the non-local Allen–Cahn equation with respect to the existence, uniqueness and matched asymptotics recovering the sharp-interface law. The numerical solution used for both approaches is based on the method of lines, with space discretization using the finite-difference method and time discretization using a higher-order time solver. In our computational studies, we investigate numerical convergence and demonstrated how the selected examples apply to various problems in natural sciences. This is a joint work with Maneesh Narayanan (Czech Technical University in Prague, Czech Republic).
10:00–10:15	*Analysis of a Numerical Scheme for the Stochastic Allen-Cahn Equation* Denisa Hanušková Czech Technical University in Prague, Czech Republic The mathematical framework of stochastic partial differential equations (SPDEs) is essential for modelling systems where random fluctuations and external noise play a crucial role. This work is concerned with numerical schemes for the stochastic Allen–Cahn equation, a non-linear reaction-diffusion equation widely used in phase-field modelling, approximations of mean curvature flow, and image segmentation. To incorporate random fluctuations, the equation is driven by a Q-Wiener process. We propose a time-discretized scheme for the stochastic Allen–Cahn equation and derive a priori estimates that allow us to prove the convergence of the scheme to analytically weak solutions. Furthermore, we introduce a fully discretized space-time numerical scheme and establish corresponding a priori estimates. The presentation will begin with a brief introduction to the main ideas behind stochastic partial differential equations, followed by the proposed schemes and the key estimates. The convergence analysis of the fully discretized scheme is left for future research.

10:15–10:30 *A Lagrange–Galerkin moving mesh method for convection–diffusion problems*

Kharisma Surya Putri
Kanazawa University, Japan

Sharp, spike-like solution profiles arising from aggregation phenomena pose a persistent challenge in the numerical simulation of convection–diffusion-type problems. Resolving such localized structures on a fixed mesh often requires excessive refinement, while maintaining stability and mass conservation remains essential for physically meaningful computations. In this talk, we present a mass-conservative Lagrange–Galerkin moving mesh method (LGMM) for convection–diffusion equations motivated by aggregation-driven applications. The method combines a characteristic-based Lagrange–Galerkin discretization with a moving mesh strategy in which the nodal points are transported according to a velocity field related to the convective flow. To prevent mesh deterioration and tangling, a diffusion- based mechanism regularizes the mesh motion, and we provide a practical time-step condition that avoids element inversion. We discuss the main analytical properties of the method, including mass conservation, stability, and error estimates that account for the time-dependent finite element space induced by the moving mesh. Particular attention is given to the role of time-dependent interpolation/projection errors, which are central to extending the analysis beyond fixed-mesh Lagrange–Galerkin schemes. Numerical experiments illustrate that LGMM can capture sharp aggregation patterns more robustly than fixed-mesh methods while preserving mass accurately, demonstrating its potential for convection-dominated and aggregation-driven simulations.

This discussion is based on [1], extended to multi-dimensional problem.

[1] Putri, K.S., Mizuochi, T., Kolbe, N., Notsu, H., *Error Estimates for First- and Second-Order Lagrange–Galerkin Moving Mesh Schemes for the One-Dimensional Convection–Diffusion Equation*, Journal of Scientific Computing 101, 2024.

10:30–10:45 *A Phase-Field Method for Hele–Shaw Flow of a Power-Law Fluid in a Time-Dependent Gap*

Kazuki Ookoma
Meiji University, Japan

This study builds on phase-field models for Hele–Shaw flows [1, 2, 3] and studies of Hele–Shaw cells with time-dependent gaps [4]. We consider a phase-field method for Hele–Shaw flow of a power-law fluid in a time-dependent gap. In such a cell, gap-averaged incompressibility gives $\nabla \cdot \mathbf{u} = -\dot{h}/h$, while the power-law rheology yields a nonlinear Darcy law of p-Laplacian type. Following the pressure–velocity formulation of Nguyen et al., we construct a two-phase diffuse-interface model in which the phase field distinguishes the two fluids, the mobility is interpolated across the diffuse interface, and the capillary pressure jump is incorporated through a phase-field curvature force. As a first step toward the full pressure-coupled model, we examine a circular-interface benchmark with a prescribed velocity field satisfying the gap-induced divergence condition. The theoretical volume conservation law gives $R(t) = R_0\sqrt{h_0/h(t)}$, which provides a direct reference solution. We compare the computed zero level set of the phase field and the conserved quantity $h(t)\int(1+\phi)/2\,dxdy$ with these theoretical values. This benchmark validates the advection and volume-conservation mechanisms before solving the nonlinear pressure equation and studying interfacial instabilities.

This work is based on [1, 2, 3, 4].

[1] S. Nguyen, R. Folch, V. K. Verma, H. Henry, and M. Plapp, *Phase-field simulations of viscous fingering in shear-thinning fluids*, Physics of Fluids, 22 (2010), 103102.

[2] R. Folch, J. Casademunt, A. Hernández-Machado, and L. Ramírez-Piscina, *Phase-field model for Hele–Shaw flows with arbitrary viscosity contrast. I. Theoretical approach*, Physical Review E, 60 (1999), 1724–1733.

[3] R. Folch, J. Casademunt, A. Hernández-Machado, and L. Ramírez-Piscina, *Phase-field model for Hele–Shaw flows with arbitrary viscosity contrast. II. Numerical study*, Physical Review E, 60 (1999), 1734–1740.

[4] A. Tatulchenkov and A. Cebers, *Magnetic fluid labyrinthine instability in Hele–Shaw cell with time dependent gap*, Physics of Fluids, 20 (2008), 054101.

10:45–11:00 Coffee Break

Session 2

11:00–11:15 *Numerical Solution of the Kuramoto-Sivashinsky Equation Using Finite Difference, Discontinuous Galerkin, and Finite Element Methods*

Miroslav Knobloch
Czech Technical University, Czech Republic

This thesis investigates the numerical solution of the Kuramoto-Sivashinsky equation, focusing initially on its one-dimensional variant under periodic boundary conditions, and subsequently expanding to the two-dimensional version subject to various boundary conditions. Chosen for its simplicity, the finite difference method using an explicit Euler scheme serves as the initial approach to solve the 1D formulation. By implementing a diverse set of initial conditions, the chaotic behavior of the equation was observed after sufficiently long integration times. To provide deeper insight into this chaotic dynamics, characteristic spatial structures, traditionally referred to as 'stripes,' are detected using pattern formation techniques. As a more advanced alternative for the 1D problem, the discontinuous Galerkin method is introduced. Although computationally and mathematically more demanding, this framework—closely related to the finite element method—provides enhanced numerical stability, thereby accommodating a wider range of initial conditions. Finally, the research transitions to the 2D task, where the weak formulation of the governing equation is rigorously derived. This problem is subsequently resolved via the finite element method, with the numerical implementation set up within the FreeFEM software environment and simulations currently underway to explore the resulting 2D dynamics.

11:15–11:30 *Long-time behavior of the dissipation function of the free energy for a nonlinear Fokker-Planck model with inhomogeneous diffusion and numerical observation*

Kouta Araki
Nihon University, Japan

We consider the long-time behavior of the dissipation function of the free energy in a nonlinear Fokker-Planck model with inhomogeneous spatial diffusion, subjected to natural boundary conditions. The model is based on a continuity equation and the law of energy dissipation associated with the free energy. For homogeneous diffusion, [1] studied the exponential decay of the dissipation function and the long-time behavior of the solution in the L^1 norm. For inhomogeneous diffusion subjected to periodic boundary conditions, the exponential decay of the dissipation function was obtained under the assumption of a large diffusion coefficient in [3]. For inhomogeneous diffusion under natural boundary conditions with porous-medium type nonlinear diffusion, a similar condition for exponential decay was obtained in [2]. By assuming that the spatial derivative of the logarithm of the diffusion coefficient is sufficiently small, we prove the exponential decay of the dissipation function. Note that this condition includes the previous conditions by [2,3]. Furthermore, we emphasize that our condition covers the classical case of homogeneous diffusion in [1] as well.

To prove the main result, we extend the entropy dissipation method to the Fokker–Planck equation with inhomogeneous spatial diffusion coefficients. To do this, it is necessary to estimate the spatial derivative of the diffusion coefficient. While previous approaches controlled this spatial derivative by assuming a large diffusion coefficient [2], our key idea is to treat it as the derivative of the log-diffusion. This change of point of view successfully covers both a small gradient and a large diffusion coefficient within a single, unified condition. Finally, we numerically examine the interaction between the derivative of the log-diffusion coefficient and the dissipation function in the one-dimensional Fokker-Planck model.

This is a joint work with Masashi, Mizuno (Nihon university, Japan).

[1] Araki, K. and Mizuno, M., *Long-time behavior of free energy in the nonlinear Fokker-Planck equation*, arXiv:2512.11455, 2025.

[2] Epshteyn, Y. and Liu, C. and Mizuno, M., *Long-time asymptotic behavior of nonlinear Fokker-Planck type equations with periodic boundary conditions*, arxiv:2404.05157, 2025.

[3] Jüngel, A., *Entropy methods for diffusive partial differential equations*, Springer, [Cham], 2016, MR3497125.

11:30–11:45 *Efficient numerical algorithm for solving c-transform on quadtree and octree meshes*

Shuhei Sasajima
Kanazawa University, Japan

The back-and-forth method can be used as a solver for partial differential equations (PDEs) that can be expressed as a gradient flow of the Wasserstein distance [1]. These include many important moving boundary problems like the porous medium equation or the Hele-Shaw problem. The c-transform $\phi^c(x) := \inf_y c(x,y) - \phi(y)$ plays an important role in the back-and-forth method [2]. In this talk, we propose an algorithm for numerically solving the c-transform on quadtree and octree meshes. The use of quadtree and octree improves computational efficiency while maintaining high accuracy by subdividing only regions which require higher resolution.

Our algorithm uses monotonicity of quadratic cost $c(x,y) = \|x-y\|/2$ and recursive way that exploits the hierarchical data structure of quadtree and octree. In practice, the computational complexity of the algorithm is $O(N \log N)$ where N is the number of nodes.

This is a joint work with Požár Norbert (Kanazawa University, Japan).

[1] Matt Jacobs, Wonjun Lee and Flavien Léger *The back-and-forth method for Wasserstein gradient flows*, ESAIM: Control, Optimisation and Calculus of Variations, 27 (2021), 28.

[2] Matt Jacobs, Flavien Léger, *A fast approach to optimal transport: the back-and-forth method*, Numerische Mathematik, (2020) 146:513-544.

11:45–12:00 *Semi-Lagrangian and fully semi-Lagrangian schemes using higher-order interpolation*

Haruki Takemura
The University of Tokyo, Japan

We establish error estimates of fully semi-Lagrangian schemes for one-dimensional nonlinear advection–diffusion equations, including Burgers' equation. In this talk, we refer to a numerical method that combines the method of characteristics with spatial interpolation simply as a semi-Lagrangian scheme. While standard semi-Lagrangian methods trace characteristic curves only to handle the advection term, the fully semi-Lagrangian technique extends this approach to both advection and diffusion terms. Although the derivation of the fully semi-Lagrangian scheme is based on a probabilistic representation of the solution, the resulting scheme is totally deterministic. This numerical method is also known as the layer method in contexts where the stochastic aspect of the scheme is emphasized. To achieve high-accuracy computations, we employ higher-order spline and Hermite interpolation operators for spatial discretization. While many studies analyze semi-Lagrangian schemes with linear interpolation, analysis for higher-order Hermite interpolation remains limited. For the fully semi-Lagrangian scheme using interpolation of degree $(2s-1)$, we establish error estimates of order $O(\Delta t + h^{2s}/\Delta t)$ in the L^2-norm, where h and Δt denote the spatial mesh and time step sizes, respectively.

This work is based on [1].

[1] H. Takemura, *Error estimates of fully semi-Lagrangian schemes for diffusive conservation laws*, arXiv:2508.03455, 2025.

12:00–14:00 Lunch Break

Session 3

14:00–14:30 *Parametric mean curvature flow for open curves and its application*

Petr Pauš
Czech Technical University in Prague, Czech Republic

The mean curvature flow given by $v = -\kappa + F$, where v denotes the normal velocity, κ the curvature, and F the forcing term, models a wide range of physical, chemical, and computational phenomena. In this presentation, we introduce the governing equation of motion, outline its fundamental mathematical properties, and focus on its numerical approximation via the parametric method. We then demonstrate the utility of this numerical approach through three distinct applications. First, we model dislocation dynamics within the crystalline structure of metals, specifically examining the Frank-Read source, precipitate-dislocation interactions, and dislocation cross-slip. Second, we analyze spiral wave dynamics in the Belousov-Zhabotinsky chemical reaction. Finally, we apply the framework to color image segmentation in computer graphics.
This work is based on [1, 2, 3].

This is a joint work with Michal Beneš, Miroslav Kolář (both Czech Technical University in Prague, Czech Rep.), and Shigetoshi Yazaki (Meiji University, Japan).

[1] Petr Pauš, Michal Beneš, *Direct approach to mean-curvature flow with topological changes.*, Kybernetika 45(4) (2009), 591–604.

[2] Petr Pauš, Shigetoshi Yazaki, *Segmentation of color images using mean curvature flow and parametric curves*, Discrete and Continuous Dynamical Systems - S 14(3) (2021), 1123-1132.

[3] Petr Pauš, Michal Beneš, Jan Kratochvíl, *A dislocation dynamics analysis of the critical cross-slip annihilation distance and the cyclic saturation stress in fcc single crystals at different temperatures.*, Acta materialia 61.20 (2013), 7917-7923.

14:30–14:45 *Some existence and regularity results for non-planar p-elasticae*

Florian Gruen
Kyoto University, Japan

In this talk we introduce the p-elastica problem in $\mathbb{R}^n$ with arbitrary $p \in (1, \infty)$ and $n \geq 2$. In particular, p-elasticae arise as critical points to the p-bending energy, the L^p norm of the curvature, under a fixed length constraint.

After some preliminary structure and regularity results (particularly the fact that completely non-planar p-elasticae are smooth and, in contrast to the planar case, do not have singularities), we show existence of closed non-planar p-elasticae and characterize them as torus knots on torii of revolution. More specifically, each (m, n)-torus knot with $m > 2n > 0$ is realized by a closed embedded p-elastica. Eventually we also analyze the global structure for varying p and provide several numerical examples.

This generalizes classical results obtained by Langer–Singer for the quadratic case $p = 2$ to general exponents p, but does not rely on explicit solution formulas for the curvature.

This work is based on [1, 2]. This is joint work with Tatsuya Miura (Kyoto University, Japan).

[1] Florian Gruen and Tatsuya Miura, *Regularity and structure of non-planar p-elasticae*, Math. Ann. 393 (2025), 2861–2897.

[2] Florian Gruen, *Existence of closed non-planar p-elasticae*, forthcoming.

14:45–15:00 *Value-Distribution-Theoretic Properties of Gauss Maps of Minimal Surfaces and Their Theoretical Background*

Shunsuke Kasao
Kanazawa University, Japan

A minimal surface is mathematically defined as a surface whose mean curvature vanishes identically, and it has been extensively studied from various perspectives as a mathematical model of soap films. One of the approaches to investigating the shape of these surfaces is to examine the unit normal vector, a mapping known as the Gauss map. For instance, a well-known result is the Osserman theorem, which states that the image of the Gauss map of a complete minimal surface in $\mathbb{R}^3$ is dense in S^2. In this presentation, building on the works of Zalcman and Ros, we provide a systematic description of the relationship among the Liouville theorem (in value distribution theory), the Montel theorem (in normal family theory), and the Bernstein theorem (in minimal surface theory). We refer to this theoretical framework as the Bloch–Ros principle. We further generalize this principle to various classes of surfaces, including maxfaces in the Lorentz–Minkowski 3-space and improper affine fronts in the affine 3-space. This work is based on [1, 2].

This is a joint work with Yu Kawakami (Kanazawa University, Japan).

[1] S. Kasao, Y. Kawakami, *Bloch-Ros principle and its application to surface theory*, Bull. Malays. Math. Sci Soc. **48** (2025), no.4, Paper No. 127, 36 pp.

[2] S. Kasao, *Application of the Bloch–Ros principle to ramification theorem*, arXiv:2605.29567 (2026).

15:00–15:15 *StableGen: From generative models to textured and animated 3D assets*

Ondřej Sakala
Faculty of Information Technology, Czech Technical University in Prague, Czech Republic

StableGen is an open-source Blender add-on that brings generative models into a familiar 3D content-creation workflow. Its original focus was coherent multi-view texturing: depth-conditioned 2D diffusion models synthesize several views of an object, which are then projected and blended on its surface. Since summer 2025, the project has grown from a texturing tool into a broader pipeline for producing usable 3D assets from a text prompt or a single reference image.

This talk follows one asset through the main stages of that pipeline. TRELLIS.2 first produces a textured mesh, after which StableGen can regenerate or refine its appearance using models such as SDXL, FLUX, and Qwen Image Edit. The earlier texturing methods will be summarized through their central ideas—geometry conditioning, visibility-aware sequential generation, weighted projection, and UV-space completion—rather than treated in full detail. The main emphasis will be on newer capabilities: estimating physically based material components such as albedo, roughness, metalness, and surface normals; transforming and combining these quantities across camera views; and baking the result for use outside the procedural setup.

For humanoid assets, a recent experimental pipeline also generates a skeleton and skinning with SkinTokens. Because the resulting bones need not have meaningful names, StableGen infers their semantic roles from geometry before retargeting existing animations. Reference-pose matching and collision-aware corrections help adapt motion to unusual generated body proportions. Visual examples will illustrate both the possibilities of this increasingly automated workflow and the failure modes that still make 3D generation a challenging applied problem.

15:15–15:30 *Coffee Break*

Session 4

15:30–15:45 *Ensemble reservoir computing for physical systems*

Yuma Nakamura
Kanazwa University, Japan

Physical computing exploits unconventional physical substrates to overcome limitations such as the high energy consumption inherent in digital computation. However, intrinsic noise and temporal fluctuations (e.g., oscillations) generally deteriorate computational performance. Here, we propose ensemble reservoir computing (ERC), a novel framework that employs ensemble averaging of spatially multiplexed systems to achieve robust information processing despite noise and temporal fluctuations. First, we prove that ensemble averaging in ERC eliminates temporal fluctuations and noise from dynamical states under certain conditions, thereby restoring computational performance to its noise-free level. Next, we show that ERC not only removes the noise and fluctuations but also actively exploits the computational capabilities that conventional reservoir computing (RC) leaves unutilized. This computational enhancement is demonstrated across diverse dynamical systems (e.g., periodic, chaotic, and strange-nonchaotic systems), in which ERC outperforms conventional RC. Finally, using energy-efficient spin-torque oscillators (STOs), we demonstrate that ERC maintains high performance even under realistic conditions, in which noise and temporal fluctuations coexist: STOs with ERC achieved 99% accuracy on an error detection test, where conventional STO reservoir with linear regression only shows a chance level performance, highlighting ERC's robustness and performance gains for physical systems.

This work is based on [1]. This is a joint work with Tomoyuki Kubota(The University of Tokyo, Japan), Yusuke Imai (The University of Tokyo, Japan), Sumito Tsunegi (National Institute of Advanced Industrial Science and Technology, Japan), Hirofumi Notsu (Kanazawa University, Japan), and Kohei Nakajima (The University of Tokyo, Japan).

[1] Yuma Nakamura, Tomoyuki Kubota, Yusuke Imai, Sumito Tsunegi, Hirofumi Notsu, and Kohei Nakajima *Ensemble Reservoir Computing for Phyisical*, arXiv:2601.21807, 2026.

15:45–16:00 *Nonlinear dynamics of quantum state distillation*

Tomáš Nguyen
Czech Technical University in Prague, Czech Republic

Quantum information science aims to exploit superposition and entanglement to perform computational tasks beyond the reach of classical methods. A major challenge in realizing such technologies is decoherence, which degrades fragile quantum states through interactions with the environment. One approach to mitigating these effects is provided by measurement-induced distillation protocols, which iteratively transform multiple imperfect copies of a quantum state into fewer copies of higher fidelity. The nonlinear nature of these protocols gives rise to rich dynamical behavior and establishes a natural connection between quantum information theory and complex dynamics. In particular, work by Kiss, Jex, and collaborators showed that certain protocols can be modeled by a one-parameter family of quadratic rational maps on the Riemann sphere. The resulting dynamical systems exhibit a variety of phenomena familiar from the theory of rational maps, including fixed points, periodic cycles, and intricate parameter dependence, while simultaneously describing the evolution of quantum states under repeated distillation. This talk introduces the mathematical framework underlying these models and discusses how tools from complex dynamics provide insight into the long-term behavior and stability of the associated quantum protocols. The presentation is intended as an accessible overview of the interplay between quantum information and dynamical systems, highlighting how ideas from one field naturally inform the other and illustrating the role of complex dynamics in the analysis of quantum information processing.
This work is based on [1, 2].

This is a joint work with HUN-REN Wigner Research Centre for Physics, Quantum Information and Complex Systems Group.

[1] L. Carleson and T. Gamelin, *Complex Dynamics*, Springer, New York, 1996.

[2] A. Portik, O. Kálmán, and T. Kiss, *Ergodic dynamics in iterated quantum protocols*, Chaos **36** (2026), 053142. https://doi.org/10.1063/5.0320372.

16:00–16:15 *Helmholtz decompositions and Laplacian isomorphisms in L^p spaces*

Md. Dalim Haque
Kanazawa University, Japan

This work is concerned with the relationship between Helmholtz decompositions and Laplacian isomorphisms in L^p spaces. The Helmholtz decomposition separates a vector field into a gradient part and a divergence-free part, and it is an important tool in the analysis of elliptic equations and fluid-type problems. This work is based on classical results on Helmholtz decompositions due to Fujiwara and Morimoto [1] and Fabes, Mendez, and Mitrea [2]. Let $1 < p < \infty$ and let $q = p/(p-1)$ be the conjugate exponent of p. Under suitable Poincaré-type assumptions, we prove that the Dirichlet Laplacian is an isomorphism if and only if the corresponding Helmholtz decompositions in both L^p and L^q hold. We also establish the analogous equivalence for the Neumann Laplacian modulo constants, where the Poincaré–Wirtinger inequality is used. The proof relies on duality arguments, adjoint isomorphisms, and the Banach–Nečas–Babuška theorem [3]. These results establish that, in this framework, solvability of the Laplacian and Helmholtz decomposability are equivalent properties.

This is a joint work with Masato Kimura (Kanazawa University, Japan), and Oussama Ounissi (Kanazawa University, Japan).

[1] Fujiwara, D., Morimoto, H., *An L^r-theorem of the Helmholtz decomposition of vector fields*, J. Fac. Sci. Univ. Tokyo Sect. IA Math., **24** (1977), 685–700.

[2] Fabes, E., Mendez, O., Mitrea, M., *Boundary layers on Sobolev-Besov spaces and Poisson's equation for the Laplacian in Lipschitz domains*, J. Funct. Anal., **159** (1998), 323–368.

[3] Saito, N., *Notes on the Banach–Nečas–Babuška theorem and Kato's minimum modulus of operators*, arXiv:1711.01533, 2018.

16:15–16:30 *Graph-based epidemic modelling with stratified time-varying urban mobility*

Kryštof Deutschar Blažek
Czech Technical University in Prague, Czech Republic

Compartmental epidemic models usually assume homogeneous mixing, whereas urban populations are redistributed between residential, work, and other activity locations throughout the day. We present an explicit metapopulation graph-based framework that couples SEIR-type reaction dynamics with empirical, time-dependent origin–destination mobility. This model was chosen over an implicit approach for the ease of modelling of potential epidemiological interventions and movement restrictions.

Starting from both a control-volume balance and a continuous-time random walk, we derive the conservative graph transport operator $-L_{\mathrm{comb}}(t)^{\mathsf{T}}$. The construction provides a discrete analogue of the advection–diffusion equation and, on regular grids with suitable edge weights, reproduces standard finite-difference Laplacians exactly. The framework is applied to hourly mobile-phone mobility data for the 25 districts of Seoul, stratified by trip purpose and age. Raw flows are normalised by reconstructed time-varying occupancies and coupled to a purpose-dependent force of infection in an SEIR system.

The numerical implementation combines a C solver core, a Cython interface, and a Python layer for data processing, graph construction, model specification, and visualisation. The Seoul application builds on [1], while the random-walk viewpoint is related to [2].

This work was carried out under the supervision of Miroslav Kolář (Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Czech Republic) and in consultation with Jooyoung Hahn (Department of Software Engineering, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Czech Republic).

[1] Y. Lim, J. Lee, and E. Jung, *Evaluating Targeted Mobility Restrictions on COVID-19 Transmission in Seoul: A Metapopulation Modeling Study Using Mobile Phone Data*, arXiv:2603.16064, 2026.

[2] N. Masuda, M. A. Porter, and R. Lambiotte, *Random Walks and Diffusion on Networks*, Physics Reports, 716–717 (2017), 1–58.

16:30–16:45 *Construction of solutions to the heat equation with a singular Neumann boundary condition*

Hinata Morooka
Kanazawa University, Japan

In this study, we consider the construction of a solution to an initial-boundary value problem for the heat equation with a Neumann boundary condition that exhibits a singularity of the form $1/|t - t_0|^\alpha$ at $t = t_0$. Our objective is to construct a solution under such a singular boundary condition. To this end, we apply the Fourier method. However, because the boundary data exhibit a strong singularity, it is necessary to modify the definition of the solution and employ a weak formulation with respect to the time variable. We also show that the parameter α must satisfy certain restrictions to guarantee the integrability of the boundary data and the convergence of the Fourier series. Nevertheless, by combining the Fourier method with a weak formulation in time, we can construct a solution under this class of singular boundary conditions by an elementary approach.

This is a joint work with Hiroshi Ohtsuka (Kanazawa University, Japan).

16:45–17:00 *Coffee Break*

Session 5

17:00–17:15 *Numerical studies of a Sobolev-gradient coupled complex boundary method for diffusion coefficient reconstruction*

Sahat Pandapotan Nainggolan
Kanazawa University, Japan

We consider the inverse problem of reconstructing spatially varying diffusion coefficients in a general elliptic equation from Cauchy data. Motivated by the domain-integral cost functional arising from the coupled complex boundary method (CCBM) [1, 2, 3], in which the diffusion coefficient does not appear explicitly, we propose a Sobolev-gradient [4] CCBM that incorporates H^1-type information into both the cost functional and the optimization procedure. The resulting formulation provides an intrinsic stabilization mechanism for the reconstruction process and is combined with Tikhonov regularization [5, 6] over a bounded admissible set of diffusion coefficients. Numerical experiments with exact and noisy measurements show that Sobolev-gradient optimization improves reconstruction quality compared with standard L^2-gradient approaches. Among the methods considered, the proposed Sobolev-gradient CCBM demonstrates the most favorable overall performance, significantly reducing oscillatory artifacts while improving robustness and sensitivity to spatial variations of the diffusion coefficient.

This is a joint work with Julius Fergy Tiongson Rabago (Kanazawa University, Japan) and Hirofumi Notsu (Kanazawa University, Japan).

[1] X. L. Cheng, R. F. Gong, W. Han, X. Zheng, *A novel coupled complex boundary method for solving inverse source problems*, Inverse Problems, (2014), Article 055002.

[2] R. Gong, X. Cheng, W. Han, *A coupled complex boundary method for an inverse conductivity problem with one measurement*, Appl. Anal., 96 (2017), 869–885.

[3] X. Zheng, X. Cheng, R. Gong, *A coupled complex boundary method for parameter identification in elliptic problems*, Int. J. Comput. Math., 97 (2020), 998–1015.

[4] J. W. Neuberger, *Sobolev Gradients and Differential Equations*, Springer, Berlin, 1997.

[5] H. W. Engl, M. Hanke, A. Neubauer, *Regularization of Inverse Problems*, Springer, 1996.

[6] A. N. Tikhonov, V. Y. Arsenin, *Solutions of Ill-posed Problems*, Wiley, New York, 1977.

17:15–17:30 *Comparison of phase-field fracture modeling with experiments in high-performance polymers*

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High-performance thermoplastic polymers are increasingly used in biomedical, aerospace, and structural applications due to their favorable strength-to-weight ratio, chemical stability, toughness, and processability. Despite these advantages, their fracture behavior remains a critical design concern, particularly when cracks initiate from notches, defects, or geometric discontinuities under load-bearing conditions. This talk presents an experimental validation of phase-field fracture modeling for high-performance engineering polymers. Standard compact tension specimens are examined under loading to evaluate the load-bearing response, crack initiation, crack propagation, and fracture resistance. The experimental results are compared with phase-field fracture simulations [1, 2], which provide a variational framework [3] for predicting damage evolution and crack growth without prescribing the crack path in advance. The discussion focuses on how measured fracture responses, including peak load and crack evolution, can be interpreted through computational modeling. Particular attention is given to the transition from stable crack growth to unstable failure and to the ability of the phase-field method to reproduce key features observed experimentally. By combining mechanical testing with mathematical modeling, this talk highlights the role of phase-field methods in bridging material experiments and predictive fracture analysis for polymer-based biomedical and engineering components.

This is a joint work with Prof. Rajesh Ghosh (IIT Mandi, India) and Dr. Himanshu Pathak (IIT Mandi, India).

[1] Francfort GA, Marigo JJ, *Revisiting brittle fracture as an energy minimization problem*, Journal of the Mechanics and Physics of Solids, 46.8 (1998), 1319-1342.

[2] Miehe C, Welschinger F, Hofacker M, *Thermodynamically consistent phase-field models of fracture: Variational principles and multi-field FE implementations*, International journal for numerical methods in engineering, 83.10 (2010), 1273-1311.

[3] Bourdin B, *Numerical implementation of the variational formulation for quasi-static brittle fracture*, Interfaces and Free Boundaries, 9.3 (2007), 411-430.

17:30–17:45 *Energy dissipation identities of fracture phase-field models*

Oussama Ounissi
Kanazawa University, Japan

Phase-field models for fracture describe crack evolution as an irreversible gradient flow of a total energy consisting of elastic and surface contributions. A fundamental property of such evolutions is energy dissipation, but establishing it rigorously is delicate: the elastic energy is defined through minimization with respect to the displacement, and differentiating it with respect to the phase field and loading data is not immediate. We address this using a continuity argument for the displacement and an envelope theorem approach, under mild regularity assumptions on the phase field and loading data. We treat both the original model and a unilateral-contact constraint variant that prevents the unphysical interpenetration of the crack lips, and illustrate the results with numerical simulations.

This is a joint work with Md Mamun Miah (Khulna University of Engineering and Technology, Bangladesh), Sayahdin Alfat (Halu Oleo University, Indonesia), and Masato Kimura (Kanazawa University, Japan).

17:45–18:00 *Two-scale modeling of self-healing in concrete: a simulation study*

Melvin Aballa Ojwok
Karlstad University, Sweden, and University of L'Aquila, Italy

We propose a microscopic reaction-diffusion model and its upscaled (macroscopic) counterpart to describe secondary concrete carbonation effects (like the carbonation of the CSH and other silicate phases) that can be linked to self-healing effects. The focus lies on solving numerically the upscaled model, aiming to quantify the capacity of the target material to self-heal. Our results show that if CO_2 is able to penetrate the cores hosting silicates, then a production of calcium carbonate (promoting self-healing) is ongoing behind the so called macroscopic carbonation front – a reaction front driven by gaseous CO_2 penetrating rapidly the material. Interestingly, since secondary carbonation effects are activated, the power-law scaling of the carbonation front approaches asymptotically $t^{\frac{2}{3}}$, thus deviating from the celebrated $t^{\frac{1}{2}}$ scaling law. The multiscale model and corresponding computational framework can be extended to include a more complex cement chemistry.

This is a joint work with Kota Kumazaki (Kyoto University of Education, Japan), and Adrian Muntean (Karlstad University, Sweden).

18:00–18:05 *Closing Remarks*

Masato Kimura, Kanazawa University, Japan